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A TABULATION OF SELECTED CONFLUENT HYPERGEOMETRIC FUNCTIONS

By

David Middleton and Virginia Johnson

January 5, 1952

Technical Report No. 140

Cruft Laboratory
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Abstract

A tabulation of the confluent hypergeometric function ${}_1F_1(\alpha;\beta;-p)$ is given for half-integral values of α(=-1/2, 1/2, 3/2,...) and integral values of β (=1,2,...10). The mesh is 0.25 for $(0 \leq p \leq 2.0)$ and 0.50 for $(2.0 < p \leq 10.0)$, and additional values for p = 20,30,....100, in steps of 10, have been computed. Accuracy of five significant figures is maintained in most instances. Specifically, the following ten tables have been prepared, with accompanying figures illustrating these functions for all $(0 \leq p \leq 9.5)$:

Table 1. ${}_1F_1(-1/2;1;-p) \longrightarrow {}_1F_1(17/2;1;-p)$, inclusive

Table 2. ${}_1F_1(-1/2;2;-p) \longrightarrow {}_1F_1(17/2;2;-p)$,

Table 3. ${}_1F_1(-1/2;3;-p) \longrightarrow {}_1F_1(19/2;3;-p)$,

Table 4. ${}_1F_1(-1/2;4;-p) \longrightarrow {}_1F_1(19/2;4;-p)$,

Table 5. ${}_1F_1(-1/2;5;-p) \longrightarrow {}_1F_1(19/2;5;-p)$,

Table 6. ${}_1F_1(-1/2;6;-p) \longrightarrow {}_1F_1(21/2;6;-p)$,

Table 7. ${}_1F_1(-1/2;7;-p) \longrightarrow {}_1F_1(21/2;7;-p)$,

Table 8. ${}_1F_1(-1/2;8;-p) \longrightarrow {}_1F_1(23/2;8;-p)$,

Table 9. ${}_1F_1(-1/2;9;-p) \longrightarrow {}_1F_1(23/2;9;-p)$,

Table 10. ${}_1F_1(-1/2;10;-p) \longrightarrow {}_1F_1(25/2;10;-p)$.

A short account of some of the more useful properties of the confluent hypergeometric function, and some of its applications in noise problems, as well as a comprehensive description of the methods of calculation, is included.

A Tabulation of Selected Confluent Hypergeometric Functions

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1. The Confluent Hypergeometric Function and Some of Its Properties:

We consider briefly in section 1 a few of the more important features of the confluent hypergeometric function, without intending a complete coverage. Further information may be obtained from the bibliography at the end of this section.

The function represented by

$$ {}_1F_1(\alpha;\beta;z) = 1+ \frac{\alpha}{\beta}\frac{z}{1!} + \frac{\alpha(\alpha+1)z^2}{\beta(\beta+1)2!} + \dots , \tag{1.1}$$

is called Kummer's function,[1] or more frequently, the confluent hypergeometric function. In Pochammer's notation we can write

$$ {}_1F_1(\alpha;\beta;z) = \sum_{n=0}^{\infty} \frac{(\alpha)_n}{(\beta)_n}\frac{z^n}{n!} , \quad \left.\begin{aligned} &(\alpha)_n = \alpha(\alpha+1)\dots(\alpha+n-1) , \quad n \geq 1 \\ &(\alpha)_0 = 1 \end{aligned}\right\} . \tag{1.2}$$

The quantity ${}_1F_1$ is an analytic function of z, which satisfies Kummer's differential equation

$$ z\frac{d^2F}{dz^2} + (\beta-z)\frac{dF}{dz} - \alpha F = 0 . \tag{1.3}$$

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1 See, for example, Magnus and Oberhettinger, Formulas and Theorems for the Special Functions of Mathematical Physics, Chelsea (New York), 1949. Chapter VI.

Equation (1.3) possesses two linearly independent solutions if $\beta \neq 0, \pm 1, \pm 2, \ldots,$ behaving simply at $z = 0$. These are

$$Z_1(z)_o = {}_1F_1(\alpha;\beta;z) = \sum_{n=o}^{\infty} \frac{(\alpha)_n}{(\beta)_n \, n!} z^n , \tag{1.4a}$$

$$Z_2(z)_o = z^{1-\beta} \, {}_1F_1(\alpha-\beta+1;2-\beta;z) . \tag{1.4b}$$

At $z \rightarrow \infty$, there are two linearly independent solutions $Z_{1,\infty}$ and $Z_{2,\infty}$, which have simple asymptotic developments, namely

$$\left.\begin{aligned} Z_1(z)_\infty &\simeq (-z)^{-\alpha} \sum_{n=o}^{\infty} \frac{(\alpha)_n(\alpha-\beta+1)_n}{n!} (-z)^{-n} , && (1.5a) \\ Z_2(z)_\infty &\simeq e^z \, z^{\alpha-\beta} \sum_{n=o}^{\infty} \frac{(\beta-\alpha)_n(1-\alpha)_n}{n!} z^{-n} , && (1.5b) \end{aligned}\right\}$$

with $-3\pi/2 < \arg z < \pi/2.$

These solutions are related to those at $z = 0$ by

$$Z_1(z)_o = e^{-\pi i\alpha} \frac{\Gamma(\beta)}{\Gamma(\beta-\alpha)} Z_1(z)_\infty + \frac{\Gamma(\beta)}{\Gamma(\alpha)} Z_2(z)_\infty , \tag{1.6a}$$

$$Z_2(z)_o = e^{-\pi i(\alpha-\beta+1)} \frac{\Gamma(2-\beta)}{\Gamma(1-\alpha)} Z_1(z)_\infty + \frac{\Gamma(2-\beta)}{\Gamma(\alpha-\beta+1)} Z_1(z)_\infty , \tag{1.6b}$$

with $-3\pi/2 < \arg z < \pi/2.$

For the functions tabulated here, β is integral, so that ${}_1F_1(\alpha;\beta;z) = Z_1(z)_o$ is the only solution of Kummer's equation.

The expressions of greatest importance in the present calculation are the recurrence relations given below in the table:

	$F_{\alpha+}$	$F_{\alpha-}$	$F_{\beta+}$	$F_{\beta-}$	F
1	α	$\alpha-\beta$			$\beta-2\alpha\mp p$
2	$\alpha\beta$		$\pm p(\beta-\alpha)$		$-\beta(\alpha\pm p)$
3	α			$1-\beta$	$\beta-\alpha-1$
4		$-\beta$	$\mp p$		β
5		$\alpha-\beta$		$\beta-1$	$1-\alpha\mp p$
6			$\pm p(\beta-\alpha)$	$\beta(\beta-1)$	$\beta(1-\beta\mp p)$

(z=p henceforth) (1.7)

The subscripts on F heading the columns indicate the addition or subtraction of unity in α or β for $F \equiv {}_1F_1(\alpha;\beta;\pm p)$; the rows list the factors by which the quantities heading the columns are to be multiplied, and the sum of each row is zero. The upper sign refers to ${}_1F_1(\alpha;\beta;p)$, while the lower sign appears for ${}_1F_1(\alpha;\beta;-p)$.

Of these recurrence relations, the following are most useful in the preparation of the tables:

(Row 1):

$$ {}_1F_1(\alpha+1;\beta;-p) = \frac{1}{\alpha}\left\{(\beta-\alpha)\,{}_1F_1(\alpha-1;\beta;-p)+(2\alpha-\beta-p)\,{}_1F_1(\alpha;\beta;-p)\right\}, \tag{1.8} $$

(Row 6):

$$ {}_1F_1(\alpha;\beta+1;-p) = \frac{1}{p(\beta-\alpha)}\left\{\beta(\beta-1)\,{}_1F_1(\alpha;\beta-1;-p)+\beta(1-\beta+p)\,{}_1F_1(\alpha;\beta;-p)\right\}, \quad (\beta\neq\alpha). \tag{1.9} $$

For large values of p it is convenient to use the asymptotic relation

$$ {}_1F_1(\alpha;\beta;-p) \backsimeq p^{-\alpha}\,\frac{\Gamma(\beta)}{\Gamma(\beta-\alpha)}\left\{1+\frac{\alpha(\alpha-\beta+1)}{p\;1!}+\frac{\alpha(\alpha+1)(\alpha-\beta+1)(\alpha-\beta+2)}{p^2\;2!}+\dots\right\} $$

$$ \mathrm{Re}(p) > 0 \tag{1.10a} $$

$$ \backsimeq p^{-\alpha}\,\Gamma(\beta)\sum_{k=0}^{\infty}\frac{(\alpha)_k(-1)^k}{k!\;p^k\,\Gamma(\beta-\alpha-k)}\;. \tag{1.10b} $$

A property also of some use is Kummer's transformation

$$ {}_1F_1(\alpha;\beta;+p) = e^p \, {}_1F_1(\beta-\alpha;\beta;-p) \ , \qquad (1.11) $$

from which functions of positive argument may be obtained.

For the values of α and β chosen here, α is half-integral, while β is an integer. Under these conditions one can use the relation[2]

$$ {}_1F_1(\alpha;2\alpha;\pm p) = 2^{2\alpha-1} \frac{\Gamma(2\alpha)}{(\pm p)^{\alpha-\frac{1}{2}}} e^{\pm p/2} \, I_{\alpha-\frac{1}{2}}(\pm p/2) \qquad (1.12) $$

to express ${}_1F_1(\alpha;\beta;-p)$ in terms of the modified Bessel functions $I_{\alpha-\frac{1}{2}}$, with the help of the recurrence relations (1.7). [For details, and the explicit representation of ${}_1F_1(\alpha;\beta;-p)$, $\alpha = -1/2, 1/2, 3/2, \ldots; \beta = 1,2,\ldots$, see the Appendix.]

Another set of confluent hypergeometric functions of importance is that for which α is, as here, half-integral, and $\beta = 1/2$ or $3/2$. These can be expressed in terms of the error function and its associated derivatives by means of the relations[3]

$$ {}_1F_1\left(\frac{2n+1}{2};1/2;-p\right) = \frac{\sqrt{2\pi}(-1)^n 2^n n!}{(2n)!} \phi^{(2n)}(\sqrt{2p}) \qquad (1.13a) $$

$$ {}_1F_1\left(\frac{2n+1}{2};3/2;-p\right) = \frac{\sqrt{2\pi}(-1)^n 2^n n!}{(2n)!} (2p)^{-\frac{1}{2}} \phi^{(2n-1)}(\sqrt{2p}) \ , \quad n=0,1,2\ldots \qquad (1.13b) $$

Here

$$ \phi^{(k)}(x) \equiv \frac{d^k}{dx^k} \frac{e^{-x^2/2}}{\sqrt{2\pi}} \quad \text{and} \quad \phi^{(-1)}(x) = \frac{1}{2} \text{Ⓗ}(x/\sqrt{2}) = \frac{1}{\sqrt{\pi}} \int_0^{x/\sqrt{2}} e^{-y^2} \, dy, \qquad (1.13c) $$

and as a special example, illustrating the case of $n < 0$, we can

2 Watson, <u>Theory of Bessel Functions</u>, Cambridge (Macmillan, New York), 1948, section 6.5.

3 See, for example, D. Middleton, "Some General Results in the Theory of Noise through Nonlinear Devices," <u>Quart. Appl. Math.</u> <u>5</u>, 445 (1948), Appendix III.

use the recurrence relation (1.8) to obtain

$$ {}_1F_1(-1/2;1/2;-p) = \sqrt{\pi p}\ \text{Ⓗ}\ (\sqrt{p}) + e^{-p} \ . \qquad (1.14)$$

Tables of $\phi^{(k)}$ have recently been completed and are now available.[4]

As a final set of functions expressible in terms of previously tabulated quantities, one has ${}_1F_1(\alpha;\beta;-p)$ where α and β are integral.[5] These can be written as polynomials in p, p^{-1}, and e^{-p}; some selected examples are included below:

$$ {}_1F_1(m;m;-p) = e^{-p} \qquad (1.15a)$$

$$ {}_1F_1(1;2;-p) = \frac{1}{p}\ (1-e^{-p}) \qquad (1.15b)$$

$$ {}_1F_1(2;3;-p) = (1-e^{-p}-pe^{-p})p^{-2} \qquad (1.15c)$$

$$ {}_1F_1(3;4;-p) = (2-2e^{-p}-2pe^{-p}-p^2e^{-p})p^{-3} \qquad (1.15d)$$

$$ {}_1F_1(2;1;-p) = (1-p)e^{-p} \qquad (1.15e)$$

$$ {}_1F_1(3;2;-p) = (1-\frac{p}{2})e^{-p} \ . \qquad (1.15f)$$

No tables of these and the associated higher-order functions have as yet been computed.

Bibliography:

1. Whittaker and Watson, Modern Analysis, Cambridge University Press, 4th ed. (1940), Chapter XVI.
2. Appel-Kampé de Fériet, Fonctions hypergéometriques and hyperspheriques. Polynomes d'Hermite, Paris (1926).

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[4] Tables of the Error Function and Its First Twenty Derivatives, Annals of the Computation Laboratory (Harvard), January 1952.

[5] D. Middleton, "The Spectrum of Frequency-Modulated Waves After Reception in Random Noise," Quart. Appl. Math. 7, 129 (1949). Appendix IV.

3. Magnus and Obserhettinger, Formulae and Theorems for the Special Functions of Mathematical Physics, Chelsea (1949).

See also the references given in (1) - (3).

2. Purpose and Application of the Tables:

Confluent hypergeometric functions of the type for which α is half-integral and β (positive) integer are of great importance in the analytical treatment of nonlinear noise problems. Specifically, for the case of a carrier, modulated or not, following rectification in normal random noise by a biased ν^{th}-law detector, one has to consider an integral of the type

$$\int_{C} z^{-1+n-\nu} e^{ib_o z-\psi z^2/2} J_m(A_o z)dz , \qquad (2.1)$$

where C is a contour along the real axis, indented downward about the singularity at $z = 0$. It has been shown that[3]

$$\int_{C} z^{\mu-1} e^{-q^2z^2} J_{\lambda}(az)dz = \frac{\pi i^{1-\lambda-\mu}(a/2q)^{\lambda}}{q^{\mu}\Gamma(\lambda+1)\Gamma(1-\frac{\lambda+\mu}{2})} \cdot {}_1F_1(\tfrac{\mu+\lambda}{2};1+\lambda;-a^2/4q^2). \qquad (2.2)$$

In the case of a half-wave linear detector, for example, b_o is zero, $\nu=1$, and $\lambda=m$, cf.(2.1), and consequently ${}_1F_1$ in (2.2) belongs to the class for which α is half-integral and β an integer. The theory of mixing of a carrier in noise and the treatment of frequency-modulated waves subject to arbitrary amounts of limiting provide numerous additional examples.[6,7] In other words, wherever Weber's first exponential integral arises,[8] we may expect to deal with a confluent hypergeometric function, and depending on parameters of the problem, α will be half-integral and β integral,

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[6] D. Middleton, Proc. I.R.E. 36, 1467 (1948); Quart. Appl. Math. 8, 59 (1950).

[7] R. A. Johnson, Doctoral Dissertation, (Harvard, 1952).

[8] Watson, Theory of Bessel Functions, Section 13.3.

the case of greatest frequency in noise problems.

Existing tables of ${}_1F_1$ are limited in order of the functions in range of the argument, and in the number of significant figures available. For this reason and because of the need for the higher-order functions in many of the applications, it is felt that a more extensive tabulation is warranted, at least until a more complete treatment, with still finer mesh, can be obtained with the help of large-scale computing machinery. A short list of previous tabulations is given below in the bibliography for this section.

Bibliography of Tables:

1. British Association for the Advancement of Science, Mathematical Tables Committee, Section A, Oxford 1926; Leeds 1927, 5 to 6-place tables of ${}_1F_1$, $-4 \leq \alpha \leq 4$, in half-units; $\beta = -1.5, -0.5, +0.5, 1.0, 1.5, 2.0, 3.0, 4.0, \ldots$
2. Jahnke and Emde, Tables of Functions (Dover, 1945), p. 275. Gives only curves for above range of α and β.
3. R. Gan Olsson, Ingenieur-Archiv 8 (1937), pp. 99-103; 4-place figures; $-0.675 < \alpha < 0.675$; $0.5 < \beta < 3$.

3. Preparation of the Tables:

The tabulations listed in the next section were obtained with the help of the series (1.2), and when the argument was large, the asymptotic expression (1.10) was used to advantage. The general procedure was to employ the available tables[9] of $I_0(x)$ and $I_1(x)$ in conjunction with the recurrence relations (1.8) and (1.9).

The principal features of the calculation can be summarized briefly (for the detailed formulae and procedures in each case, see the Appendix):

(i) The values of ${}_1F_1$ are in every case accurate to the number of significant figures given, usually five.

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[9] British Association for the Advancement of Science, Mathematical Tables. See also reference 8.

(ii) Those values of ${}_1F_1$ which were computed with the help of the general series, and the formulae involving I_0 and I_1 are accurate for more than five significant figures, e.g., seven and eight figures. Tables of greater accuracy are given in the original computations.

(iii) Similarly, ${}_1F_1$ computed with the aid of the recurrence relation (1.8) have been found to seven and eight-figure accuracy, for $0 \leq p \leq 10$.

(iv) For p = 20, 30, 40, and 50, the number of significant figures is reduced from five, and in some cases it was not possible to obtain any reliable value, due to the slow "semi-convergence" of the asymptotic formula (1.10). In the great majority of the cases, however, seven and eight-figure accuracy was originally obtained.

We have observed that

(v) the general series

$$ {}_1F_1(\alpha;\beta;\pm p) = 1 + \frac{\alpha(\pm p)}{\beta\ 1!} + \frac{\alpha(\alpha+1)(\pm p)^2}{\beta(\beta+1)2!} + \ldots $$

can be conveniently used when $0 \leq p \leq 1.0$, $1.0 < \beta < 8.0$, and $0 \leq p \leq 2.0$; $\beta \geq 9.0$.

(vi) the asymptotic formula (1.10) is, in general, satisfactory for $p \geq 10$, and is more accurate for the larger values of β.

(vii) the recurrence relation (1.8) proves good for the higher values of α, and serves well for checking results obtained directly by the formulae.

A brief word is now necessary concerning the procedures by which the results were checked. The principal method consisted in using various recurrence relations, cf. (1.7), in the following way:

(viii) every table, computed by the explicit relations

[cf. Appendix] involving I_o and I_1 were recomputed with the help of (1.8), and at least one or all of the recurrence formulae (1.9), and the following expressions, derived from (1.7):

$$ {}_1F_1(\alpha+1;\beta+1;-p) = \frac{\beta}{p\alpha}\left\{-(\beta-1)\,{}_1F_1(\alpha-1;\beta-1;-p) + (\beta-1+p)\,{}_1F_1(\alpha;\beta;-p)\right\} \tag{3.1} $$

$$ {}_1F_1(\alpha;\beta+1;-p) = \frac{\beta(1-\beta)}{(\beta-\alpha)p}\left\{{}_1F_1(\alpha;\beta;-p) - {}_1F_1(\alpha-1;\beta-1;-p)\right\}, \tag{3.2} $$

(ix) tables of ${}_1F_1$, determined from the general series, were recalculated from (1.8), in some cases, where possible, from the explicit formulae in I_o and I_1, and with the help of

$$ {}_1F_1(\alpha+1;\beta+1;-p) = \frac{\beta}{p\alpha}[-(\beta-1)\,{}_1F_1(\alpha-1;\beta-1;-p) + (\beta-1+p)\,{}_1F_1(\alpha;\beta;-p)]. \tag{3.3} $$

(x) tables of ${}_1F_1$ computed with the aid of (1.8), were redone using (1.9), (3.3), and in some cases

$$ {}_1F_1(\alpha;\beta+1;-p) = \frac{\beta(1-\beta)}{(\beta-\alpha)p}[{}_1F_1(\alpha;\beta;-p) - {}_1F_1(\alpha-1;\beta-1;-p)]. \tag{3.4} $$

(xi) tables of ${}_1F_1$ determined by the asymptotic formula were checked with the help of (1.8), and (3.3) in some cases. Whenever practicable, the asymptotic expression was used to compute ${}_1F_1(\alpha;\beta;-10)$, serving as an additional check for this case, since for every α,β considered here ${}_1F_1(\alpha;\beta;-10)$ was also computed by the direct expressions.

Figures 1-11 illustrate the ten tables, which follow.

Table 1. $\alpha = -1/2 \longrightarrow 17/2;\ \beta = 1.$

p	${}_1F_1(-\frac{1}{2};1;-p)$	${}_1F_1(\frac{1}{2};1;-p)$	${}_1F_1(\frac{3}{2};1;-p)$	${}_1F_1(\frac{5}{2};1;-p)$	${}_1F_1(\frac{7}{2};1;-p)$
0	1.0	1.0	1.0	1.0	1.0
.25	1.12125	.88595	.67828	+.49601	+.33704
.50	1.23558	.79102	.44456	+.18089	-.013489
.75	1.34377	.71166	.27628	-.0069923	-.17486
1.00	1.44649	.64504	.15642	-.11073	-.22673
1.25	1.54432	.58882	.072258	-.16014	-.21951
1.50	1.63774	.54116	+.014249	-.17564	-.18419
1.75	1.72721	.50055	-.024720	-.17097	-.13904
2.00	1.81310	.46576	-.049939	-.15525	-.094239
2.50	1.97540	.40984	-.073775	-.11202	-.022947
3.00	2.12685	.36743	-.077749	-.070645	+.018391
3.50	2.26914	.33456	-.072768	-.038752	.035910
4.00	2.40362	.30851	-.064448	-.016906	.038669
4.50	2.53135	.28743	-.055537	-.0032490	.033972
5.00	2.65320	.27005	-.047262	+.0045096	.026554
5.50	2.76988	.25545	-.040073	.0083545	.019031
6.00	2.88196	.24300	-.034041	.0097772	.012603
6.50	2.98994	.23223	-.029073	.0098079	.0076358
7.00	3.09422	.22280	-.025015	.0091152	.0040707
7.50	3.19515	.21446	-.021708	.0081091	.0016718
8.00	3.29302	.20700	-.019007	.0070263	+.00016198
8.50	3.38810	.20029	-.016790	.0059924	-.00071251
9.00	3.48061	.19420	-.014957	.0050657	-.0011574
9.50	3.57074	.18864	-.013429	.0042658	-.0013273
10.00	3.65867	.18354	-.012145	.0035912	-.0013320
20.00	5.10975	.12783	-.0035798	.0003462	$-.681 \cdot 10^{-4}$
30.00	6.23211	.10390	-.0018612	.00010849	$-.11584 \cdot 10^{-4}$
40.00	7.18124	.089780	-.0011833	$.49559 \cdot 10^{-4}$	$-.36865 \cdot 10^{-5}$
50.00	8.01884	.080197	-.00083624	$.27354 \cdot 10^{-4}$	$-.15647 \cdot 10^{-5}$
60.00	8.77688	.073146	-.00063101	$.16937 \cdot 10^{-4}$	$-.78759 \cdot 10^{-6}$
70.00	9.47448	.067678	-.00049788	$.11332 \cdot 10^{-4}$	$-.44405 \cdot 10^{-6}$
80.00	10.12412	.063278	-.00040578	$.80178 \cdot 10^{-5}$	$-.27150 \cdot 10^{-6}$
90.00	10.73452	.059638	-.00033895	$.59171 \cdot 10^{-5}$	$-.17642 \cdot 10^{-6}$
100.00	11.31204	.056562	-.00028865	$.45133 \cdot 10^{-5}$	$-.12020 \cdot 10^{-6}$

Table 1. (Continued)

p	${}_1F_1(\frac{9}{2};1;-p)$	${}_1F_1(\frac{11}{2};1;-p)$	${}_1F_1(\frac{13}{2};1;-p)$	${}_1F_1(\frac{15}{2};1;-p)$	${}_1F_1(\frac{17}{2};1;-p$
0	1.0	1.0	1.0	1.0	1.0
.25	+.19942	+.081309	-.019028	-.10320	-.17270
.50	-.15041	-.24018	-.29181	-.31304	-.31057
.75	-.25729	-.27852	-.25791	-.21072	-.14874
1.00	-.24481	-.20446	-.13428	-.054238	+.022365
1.25	-.18352	-.10455	-.016176	+.061714	+.11893
1.50	-.11136	-.017592	+.063923	.11814	+.14151
1.75	-.046715	+.043262	.10311	.12600	+.11643
2.00	+.0031931	.077555	.11019	.10391	+.070749
2.50	.057067	.087597	.072759	+.032219	-.013654
3.00	.066225	.059279	+.021262	-.020720	-.048816
3.50	.053330	.025400	-.013616	-.039297	-.043216
4.00	.034172	+.00029937	-.027632	-.034262	-.021735
4.50	.016880	-.013294	-.027105	-.020026	-.0018757
5.00	+.0043656	-.017742	-.019701	-.0062040	+.0096297
5.50	-.0032487	-.016607	-.010929	+.0031226	.013011
6.00	-.0069837	-.012906	-.0036725	.0075308	.011216
6.50	-.0080964	-.0086378	+.0011276	.0082630	.0072858
7.00	-.0076739	-.0048714	.0036215	.0069077	.0033086
7.50	-.0065087	-.0020235	.0044056	.0047622	+.00030909
8.00	-.0051113	-.00012598	.0041362	.0026519	-.0014631
8.50	-.0037713	+.00097321	.0033510	+.00098093	-.0021849
9.00	-.0026263	.0014838	.0024186	-.00013926	-.0021890
9.50	-.0017197	.0016056	.0015530	-.00076124	-.0018027
10.00	-.0010428	+.0014995	.00085322	-.0010063	-.0012761
20.00					
30.00	$+.194 \cdot 10^{-5}$	$-.48 \cdot 10^{-6}$	$.2 \cdot 10^{-6}$		
40.00	$.41225 \cdot 10^{-6}$	$-.6432 \cdot 10^{-7}$	$.135 \cdot 10^{-7}$	$-.38 \cdot 10^{-8}$	$+.1 \cdot 10^{-8}$
50.00	$.13202 \cdot 10^{-6}$	$-.15164 \cdot 10^{-7}$	$.22684 \cdot 10^{-8}$	$-.431 \cdot 10^{-9}$	$.103 \cdot 10^{-9}$
60.00	$.53432 \cdot 10^{-7}$	$-.48716 \cdot 10^{-8}$	$.56942 \cdot 10^{-9}$	$-.82864 \cdot 10^{-10}$	$.14736 \cdot 10^{-10}$
70.00	$.25206 \cdot 10^{-7}$	$-.19072 \cdot 10^{-8}$	$.18329 \cdot 10^{-9}$	$-.21689 \cdot 10^{-10}$	$.30950 \cdot 10^{-11}$
80.00	$.13251 \cdot 10^{-7}$	$-.85738 \cdot 10^{-9}$	$.70015 \cdot 10^{-10}$	$-.69903 \cdot 10^{-11}$	$.83477 \cdot 10^{-12}$
90.00	$.75535 \cdot 10^{-8}$	$-.42697 \cdot 10^{-9}$	$.30325 \cdot 10^{-10}$	$-.26202 \cdot 10^{-11}$	$.26928 \cdot 10^{-12}$
100.00	$.45842 \cdot 10^{-8}$	$-.23010 \cdot 10^{-9}$	$.14462 \cdot 10^{-10}$	$-.11018 \cdot 10^{-11}$	$.99429 \cdot 10^{-13}$

Table 2. $\alpha = -1/2 \longrightarrow 17/2;\ \beta = 2.$

p	${}_1F_1(-\frac{1}{2};2;-p)$	${}_1F_1(\frac{1}{2};2;-p)$	${}_1F_1(\frac{3}{2};2;-p)$	${}_1F_1(\frac{5}{2};2;-p)$
0	1.0	1.0	1.0	1.0
.25	1.06124	.94121	.83068	.72908
.50	1.12010	.88913	.69290	.52734
.75	1.17678	.84281	.58052	.37769
1.00	1.23148	.80146	.48861	.26715
1.25	1.28434	.76439	.41325	.18592
1.50	1.33551	.73105	.35128	.12659
1.75	1.38512	.70095	.30016	.083572
2.00	1.43329	.67367	.25785	.052657
2.50	1.52568	.62623	.19344	+.015298
3.00	1.61339	.58647	.14839	-.0023680
3.50	1.69701	.55274	.11638	-.0097188
4.00	1.77700	.52378	.093239	-.011886
4.50	1.85378	.49865	.076215	-.011620
5.00	1.92768	.47663	.063462	-.010354
5.50	1.99898	.45717	.053732	-.0088051
6.00	2.06792	.43983	.046174	-.0073031
6.50	2.13471	.42426	.040201	-.0059816
7.00	2.19955	.41020	.035402	-.0048757
7.50	2.26257	.39742	.031489	-.0039756
8.00	2.32393	.38575	.028251	-.0032541
8.50	2.38375	.37504	.025538	-.0026802
9.00	2.44213	.36516	.023239	-.0022247
9.50	2.49917	.35601	.021270	-.0018626
10.00	2.55495	.34751	.019568	-.0015736
20.00	3.48953	.24910	.0065706	-.00019630
30.00	4.22283	.20427	.0035254	$-.65655\cdot10^{-4}$
40.00	4.84659	.17729	.0022741	$-.30821\cdot10^{-4}$
50.00	5.39882	.15877	.0016207	$-.17272\cdot10^{-4}$
60.00	5.89961	.14506	.0012296	$-.10799\cdot10^{-4}$
70.00	6.36111	.13438	.00097395	$-.72745\cdot10^{-5}$
80.00	6.79133	.12576	.00079605	$-.51725\cdot10^{-5}$
90.00	7.19588	.11861	.00066641	$-.38319\cdot10^{-5}$
100.00	7.57888	.11255	.00056850	$-.29316\cdot10^{-5}$

Table 2. (Cont.) $\alpha = -1/2 \longrightarrow 17/2; \beta = 2.$

p	${}_1F_1(\frac{7}{2};2;-p)$	${}_1F_1(\frac{9}{2};2;-p)$	${}_1F_1(\frac{11}{2};2;-p)$
0	1.0	1.0	1.0
.25	.63585	.55048	.47246
.50	.38876	.27383	.17956
.75	.22382	.10991	+.028311
1.00	.11600	+.018076	-.040342
1.25	.047495	-.028793	-.063177
1.50	+.0056996	-.048554	-.062510
1.75	-.018245	-.052758	-.051415
2.00	-.030507	-.048716	-.037181
2.50	-.035629	-.032006	-.012212
3.00	-.029679	-.015944	+.0023154
3.50	-.021332	-.0049771	.0079800
4.00	-.013894	+.0011242	.0084681
4.50	-.0082713	.0037982	.0067053
5.00	-.0044088	.0044376	.0044216
5.50	-.0019412	.0040509	.0024288
6.00	-.00047099	.0032645	.00098710
6.50	+.00033416	.0024203	$+.83282 \cdot 10^{-4}$
7.00	.00072064	.0016778	-.00040036
7.50	.00085831	.0010907	-.00059803
8.00	.00085804	.00065916	-.00062317
8.50	.00078881	.00035986	-.00055818
9.00	.00069146	.00016322	-.00045668
9.50	.00058875	$+.41312 \cdot 10^{-4}$	-.00035003
10.00	.00049232	$-.28920 \cdot 10^{-4}$	-.00025423
20.00	$.207 \cdot 10^{-4}$	$-.4 \cdot 10^{-5}$	
30.00	$.40024 \cdot 10^{-5}$	$-.451 \cdot 10^{-6}$	$.810 \cdot 10^{-7}$
40.00	$.13311 \cdot 10^{-5}$	$-.10247 \cdot 10^{-6}$	$.11914 \cdot 10^{-7}$
50.00	$.57837 \cdot 10^{-6}$	$-.33934 \cdot 10^{-7}$	$.29436 \cdot 10^{-8}$
60.00	$.29541 \cdot 10^{-6}$	$-.14017 \cdot 10^{-7}$	$.97173 \cdot 10^{-9}$
70.00	$.16823 \cdot 10^{-6}$	$-.67036 \cdot 10^{-8}$	$.38733 \cdot 10^{-9}$
80.00	$.10362 \cdot 10^{-6}$	$-.35594 \cdot 10^{-8}$	$.17636 \cdot 10^{-9}$
90.00	$.67706 \cdot 10^{-7}$	$-.20441 \cdot 10^{-8}$	$.88672 \cdot 10^{-10}$
100.00	$.46335 \cdot 10^{-7}$	$-.12479 \cdot 10^{-8}$	$.48143 \cdot 10^{-10}$

Table 2. (Cont.) $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 2$.

p	${}_1F_1(\frac{13}{2};2;-p)$	${}_1F_1(\frac{15}{2};2;-p)$	${}_1F_1(\frac{17}{2};2;-p)$
0	1.0	1.0	1.0
.25	.40135	.33667	+.27802
.50	+.10324	+.042465	-.0049357
.75	-.027477	-.062929	-.082634
1.00	-.070182	-.080044	-.076603
1.25	-.070700	-.062312	-.045775
1.50	-.054343	-.036148	-.015576
1.75	-.034201	-.013076	+.0054673
2.00	-.016320	+.0031439	.016579
2.50	+.0059352	.016216	.018350
3.00	.012672	.013994	.0093653
3.50	.011147	.0073376	+.0011196
4.00	.0069829	+.0016575	-.0031318
4.50	.0030691	-.0015730	-.0040334
5.00	+.00039179	-.0026995	-.0031667
5.50	-.0010323	-.0025549	-.0017979
6.00	-.0015390	-.0018672	-.00061414
6.50	-.0015024	-.0010978	+.00015035
7.00	-.0012133	-.00046946	.00051416
7.50	-.00085721	$-.47552 \cdot 10^{-4}$	.00059375
8.00	-.00053277	+.00018553	.00051439
8.50	-.00027975	.00027884	.00037245
9.00	-.00010386	.00028421	.00022774
9.50	$+.55317 \cdot 10^{-5}$	.00024361	.00010963
10.00	$+.64627 \cdot 10^{-4}$	.00018595	$+.26986 \cdot 10^{-4}$
20.00			
30.00	$-.2 \cdot 10^{-7}$	$.9 \cdot 10^{-8}$	
40.00	$-.1946 \cdot 10^{-8}$	$.433 \cdot 10^{-9}$	$-.13 \cdot 10^{-9}$
50.00	$-.34864 \cdot 10^{-9}$	$.53989 \cdot 10^{-10}$	$-.10675 \cdot 10^{-10}$
60.00	$-.90683 \cdot 10^{-10}$	$.10871 \cdot 10^{-10}$	$-.16267 \cdot 10^{-11}$
70.00	$-.29865 \cdot 10^{-10}$	$.29282 \cdot 10^{-11}$	$-.35406 \cdot 10^{-12}$
80.00	$-.11592 \cdot 10^{-10}$	$.96257 \cdot 10^{-12}$	$-.97813 \cdot 10^{-13}$
90.00	$-.50811 \cdot 10^{-11}$	$.36606 \cdot 10^{-12}$	$-.32105 \cdot 10^{-13}$
100.00	$-.24456 \cdot 10^{-11}$	$.15564 \cdot 10^{-12}$	$-.12012 \cdot 10^{-13}$

Table 3. $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 3$.

p	${}_1F_1(-\frac{1}{2};3;-p)$	${}_1F_1(\frac{1}{2};3;-p)$	${}_1F_1(\frac{3}{2};3;-p)$	${}_1F_1(\frac{5}{2};3;-p)$
0	1.0	1.0	1.0	1.0
.25	1.04103	.96021	.88422	.81284
.50	1.08085	.92387	.78490	.66224
.75	1.11955	.89060	.69944	.54088
1.00	1.15719	.86005	.62568	.44292
1.25	1.19386	.83192	.56183	.36373
1.50	1.22960	.80595	.50637	.29958
1.75	1.26448	.78192	.45805	.24752
2.00	1.29856	.75962	.41582	.20519
2.50	1.36446	.71956	.34623	.14252
3.00	1.42764	.68461	.29205	.10051
3.50	1.48838	.65387	.24935	.072056
4.00	1.54693	.62661	.21527	.052562
4.50	1.60348	.60228	.18775	.039038
5.00	1.65823	.58042	.16527	.029526
5.50	1.71131	.56066	.14670	.022740
6.00	1.76287	.54270	.13122	.017826
6.50	1.81303	.52629	.11817	.014210
7.00	1.86188	.51124	.10708	.011508
7.50	1.90953	.49737	.097583	.0094571
8.00	1.95606	.48454	.089375	.0078763
8.50	2.00153	.47264	.082235	.0066397
9.00	2.04601	.46155	.075982	.0056587
9.50	2.08957	.45119	.070472	.0048701
10.00	2.13226	.44149	.065589	.0042284
20.00	2.85644	.32404	.024252	.00067670
30.00	3.43185	.26790	.013383	.00023940
40.00	3.92396	.23346	.0087506	.00011524
50.00	4.36098	.20960	.0062861	$.65517 \cdot 10^{-4}$
60.00	4.75805	.19182	.0047944	$.41347 \cdot 10^{-4}$
70.00	5.12447	.17791	.0038117	$.28035 \cdot 10^{-4}$
80.00	5.46639	.16664	.0031241	$.20030 \cdot 10^{-4}$
90.00	5.78816	.15727	.0026210	$.14894 \cdot 10^{-4}$
100.00	6.09297	.14933	.0022397	$.11429 \cdot 10^{-4}$

Table 3. (Cont.) $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 3$.

p	${}_1F_1(\frac{7}{2};3;-p)$	${}_1F_1(\frac{9}{2};3;-p)$	${}_1F_1(\frac{11}{2};3;-p)$
0	1.0	1.0	1.0
.25	.74583	.68298	.62409
.50	.55432	.45972	.37710
.75	.41033	.30375	.21760
1.00	.30231	.19584	.11684
1.25	.22148	.12206	.055015
1.50	.16119	.072338	+.018608
1.75	.11636	.039444	-.0015349
2.00	.083164	+.018209	-.011535
2.50	.040742	-.0028987	-.015835
3.00	.018207	-.0091562	- 012173
3.50	.0066361	-.0093457	-.0074041
4.00	+.0010040	-.0075089	-.0036720
4.50	-.0014881	-.0053642	-.0012920
5.00	-.0023782	-.0035386	$+.63956\cdot10^{-5}$
5.50	-.0024959	-.0021790	.00058987
6.00	-.0022774	-.0012452	.00075912
6.50	-.0019433	-.00064190	.00071910
7.00	-.0015990	-.00027347	.00059376
7.50	-.0012890	$-.61982\cdot10^{-4}$	.00045034
8.00	-.0010280	$+.49719\cdot10^{-4}$	.00032058
8.50	-.00081624	.00010093	.00021601
9.00	-.00064804	.00011739	.00013776
9.50	-.00051608	.00011525	$.82388\cdot10^{-4}$
10.00	-.00041318	.00010425	$.45062\cdot10^{-4}$
20.00	$-.2170\cdot10^{-4}$	$.25\cdot10^{-5}$	$-.6\cdot10^{-6}$
30.00	$-.46438\cdot10^{-5}$	$.29689\cdot10^{-6}$	$-.354\cdot10^{-7}$
40.00	$-.16076\cdot10^{-5}$	$.71680\cdot10^{-7}$	$-.57192\cdot10^{-8}$
50.00	$-.71401\cdot10^{-6}$	$.24492\cdot10^{-7}$	$-.14751\cdot10^{-8}$
60.00	$-.36982\cdot10^{-6}$	$.10314\cdot10^{-7}$	$-.49962\cdot10^{-9}$
70.00	$-.21265\cdot10^{-6}$	$.49982\cdot10^{-8}$	$-.20260\cdot10^{-9}$
80.00	$-.13190\cdot10^{-6}$	$.26794\cdot10^{-8}$	$-.93394\cdot10^{-10}$
90.00	$-.86658\cdot10^{-7}$	$.15500\cdot10^{-8}$	$-.47396\cdot10^{-10}$
100.00	$-.59559\cdot10^{-7}$	$.95165\cdot10^{-9}$	$-.25921\cdot10^{-10}$

Table 3. (Cont.) $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 3$.

p	${}_1F_1(\frac{13}{2};3;-p)$	${}_1F_1(\frac{15}{2};3;-p)$	${}_1F_1(\frac{17}{2};3;-p)$
0	1.0	1.0	1.0
.25	.56896	.51738	.46919
.50	.30527	.24311	.18960
.75	.14877	.094538	+.052547
1.00	.059681	+.019723	-.0068816
1.25	+.012036	-.013421	-.026459
1.50	-.010889	-.024260	-.027430
1.75	-.019673	-.024143	-.021192
2.00	-.020861	-.019464	-.013435
2.50	-.014518	-.0082244	-.0017070
3.00	-.0069046	-.00088096	+.0030856
3.50	-.0018098	+.0021770	.0035531
4.00	+.00074261	.0026627	.0023946
4.50	.0016161	.0020632	.0010935
5.00	.0016119	.0012365	+.00018691
5.50	.0012586	.00055369	-.00027527
6.00	.00084202	+.00010941	-.00041769
6.50	.00048789	-.00012450	-.00038403
7.00	.00023226	-.00021252	-.00028104
7.50	$+.69114 \cdot 10^{-4}$	-.00021591	-.00017101
8.00	$-.22599 \cdot 10^{-4}$	-.00017958	$-.82214 \cdot 10^{-4}$
8.50	$-.65514 \cdot 10^{-4}$	-.00013143	$-.22026 \cdot 10^{-4}$
9.00	$-.78404 \cdot 10^{-4}$	$-.86238 \cdot 10^{-4}$	$+.12547 \cdot 10^{-4}$
9.50	$-.74856 \cdot 10^{-4}$	$-.50121 \cdot 10^{-4}$	$.28206 \cdot 10^{-4}$
10.00	$-.63771 \cdot 10^{-4}$	$-.24264 \cdot 10^{-4}$	$.31792 \cdot 10^{-4}$
20.00			
30.00	$.69[illegible] \cdot 10^{-8}$	$-.2[illegible] \cdot 10^{-8}$	$.1[illegible] \cdot 10^{-8}$
40.00	$.69300 \cdot 10^{-9}$	$-.119[illegible] \cdot 10^{-9}$	$.282 \cdot 10^{-10}$
50.00	$.13169 \cdot 10^{-9}$	$-.16105 \cdot 10^{-10}$	$.25865 \cdot 10^{-11}$
60.00	$.35414 \cdot 10^{-10}$	$-.33851 \cdot 10^{-11}$	$.41660 \cdot 10^{-12}$
70.00	$.11920 \cdot 10^{-10}$	$-.93694 \cdot 10^{-12}$	$.93780 \cdot 10^{-13}$
80.00	$.46988 \cdot 10^{-11}$	$-.31388 \cdot 10^{-12}$	$.26510 \cdot 10^{-13}$
90.00	$.20834 \cdot 10^{-11}$	$-.12105 \cdot 10^{-12}$	$.88481 \cdot 10^{-14}$
100.00	$.10118 \cdot 10^{-11}$	$-.52024 \cdot 10^{-13}$	$.33531 \cdot 10^{-14}$

Table 4. $\alpha = -1/2 \longrightarrow 19/2$; $\beta = 4$.

p	${}_1F_1(-\frac{1}{2};4;-p)$	${}_1F_1(\frac{1}{2};4;-p)$	${}_1F_1(\frac{3}{2};4;-p)$	${}_1F_1(\frac{5}{2};4;-p)$
0	1.0	1.0	1.0	1.0
.25	1.03087	.96988	.91184	.85661
.50	1.06100	.94188	.83383	.73597
.75	1.09044	.91579	.76463	.63425
1.00	1.11923	.89144	.70309	.54827
1.25	1.14740	.86866	.64822	.47544
1.50	1.17499	.84730	.59916	.41357
1.75	1.20202	.82726	.55520	.36090
2.00	1.22853	.80840	.51570	.31594
2.50	1.28009	.77387	.44800	.24445
3.00	1.32983	.74302	.39256	.19154
3.50	1.37794	.71530	.34673	.15196
4.00	1.42454	.69023	.30851	.12203
4.50	1.46977	.66747	.27636	.099140
5.00	1.51372	.64668	.24909	.081445
5.50	1.55650	.62763	.22579	.067617
6.00	1.59819	.61009	.20574	.056696
6.50	1.63886	.59388	.18836	.047983
7.00	1.67859	.57885	.17321	.040962
7.50	1.71744	.56486	.15992	.035250
8.00	1.75545	.55182	.14819	.030562
8.50	1.79268	.53961	.13779	.026681
9.00	1.82917	.52815	.12852	.023441
9.50	1.86497	.51738	.12023	.020716
10.00	1.90011	.50723	.11277	.018408
20.00	2.50264	.37986	.044969	.0035364
30.00	2.98678	.31639	.025452	.0013144
40.00	3.40294	.27679	.016854	.00064765
50.00	3.77356	.24908	.012199	.00037323
60.00	4.11094	.22831	.0093512	.00023765
70.00	4.42269	.21200	.0074612	.00016216
80.00	4.71387	.19874	.0061318	.00011640
90.00	4.98810	.18770	.0051551	$.86869 \cdot 10^{-4}$
100.00	5.24802	.17831	.0044126	$.66849 \cdot 10^{-4}$

Table 4. (Cont.) $\alpha = -1/2 \longrightarrow 19/2$; $\beta = 4$.

p	${}_1F_1(\frac{7}{2};4;p)$	${}_1F_1(\frac{9}{2};4;p)$	${}_1F_1(\frac{11}{2};4;p)$	${}_1F_1(\frac{13}{2};4;p)$
0	1.0	1.0	1.0	1.0
.25	.80408	.75415	.70671	.66164
.50	.64749	.56763	.49569	.43101
.75	.52220	.42631	.34460	.27533
1.00	.42185	.31938	.23702	.17147
1.25	.34139	.23861	.16091	.10315
1.50	.27678	.17770	.10746	.058995
1.75	.22485	.13186	.070249	.031094
2.00	.18304	.097432	.044617	+.013988
2.50	.12213	.052369	.015524	-.0015811
3.00	.082301	.027363	+.0030170	-.0052686
3.50	.056074	.013699	-.0016642	-.0047950
4.00	.038669	.0063847	-.0028777	-.0033109
4.50	.027017	.0025841	-.0027148	-.0019387
5.00	.019143	+.00069620	-.0021270	-.00096332
5.50	.013765	-.00017289	-.0015103	-.00036474
6.00	.010051	-.00051611	-.0010021	$-.41450 \cdot 10^{-4}$
6.50	.0074554	-.00060066	-.00062815	+.00010671
7.00	.0056173	-.00056806	-.00037167	.00015493
7.50	.0042985	-.00049082	-.00020493	.00015249
8.00	.0033391	-.00040416	-.00010157	.00012869
8.50	.0026315	-.00032371	$-.40617 \cdot 10^{-4}$	$.99362 \cdot 10^{-4}$
9.00	.0021022	-.00025514	$-.67898 \cdot 10^{-5}$	$.72053 \cdot 10^{-4}$
9.50	.0017009	-.00019937	$+.10377 \cdot 10^{-4}$	$.49656 \cdot 10^{-4}$
10.00	.0013925	-.00015523	$.17756 \cdot 10^{-4}$	$+.32650 \cdot 10^{-4}$
20.00	.00010476	$-.363 \cdot 10^{-5}$	$.48 \cdot 10^{-6}$	
30.00	$.24404 \cdot 10^{-4}$	$-.49407 \cdot 10^{-6}$	$.3324 \cdot 10^{-7}$	$-.424 \cdot 10^{-8}$
40.00	$.87640 \cdot 10^{-5}$	$-.12595 \cdot 10^{-6}$	$.58050 \cdot 10^{-8}$	$-.48091 \cdot 10^{-9}$
50.00	$.39739 \cdot 10^{-5}$	$-.44310 \cdot 10^{-7}$	$.15580 \cdot 10^{-8}$	$-.96408 \cdot 10^{-10}$
60.00	$.20858 \cdot 10^{-5}$	$-.19006 \cdot 10^{-7}$	$.54070 \cdot 10^{-9}$	$-.26752 \cdot 10^{-10}$
70.00	$.12106 \cdot 10^{-5}$	$-.93278 \cdot 10^{-8}$	$.22289 \cdot 10^{-9}$	$-.91937 \cdot 10^{-11}$
80.00	$.75609 \cdot 10^{-6}$	$-.50469 \cdot 10^{-8}$	$.10398 \cdot 10^{-9}$	$-.36785 \cdot 10^{-11}$
90.00	$.49936 \cdot 10^{-6}$	$-.29403 \cdot 10^{-8}$	$.53246 \cdot 10^{-10}$	$-.16493 \cdot 10^{-11}$
100.00	$.34465 \cdot 10^{-6}$	$-.18153 \cdot 10^{-8}$	$.29327 \cdot 10^{-10}$	$-.80797 \cdot 10^{-12}$

Table 4. (Cont.) $\alpha = -1/2 \longrightarrow 19/2$; $\beta = 4$.

p	${}_1F_1(\frac{15}{2};4;-p)$	${}_1F_1(\frac{17}{2};4;-p)$	${}_1F_1(\frac{19}{2};4;-p)$
0	1.0	1.0	1.0
.25	.61886	.57827	.53977
.50	.37297	.32103	.27464
.75	.21692	.16796	.12723
1.00	.11987	.079813	.049215
1.25	.061097	.031290	+.010908
1.50	.026741	+.0063406	-.0055784
1.75	+.0076631	-.0050595	-.010753
2.00	-.0020959	-.0090430	-.010593
2.50	-.0075518	-.0078209	-.0056630
3.00	-.0060237	-.0039666	-.0014776
3.50	-.0034173	-.0011796	+.00049079
4.00	-.0014401	+.00020104	.00097526
4.50	-.00029806	.00064643	.00080423
5.00	+.00022526	.00062976	.00047346
5.50	.00038448	.00045216	.00019542
6.00	.00036631	.00026355	$+.23112 \cdot 10^{-4}$
6.50	.00028264	.00011979	$-.58032 \cdot 10^{-4}$
7.00	.00019062	$+.29365 \cdot 10^{-4}$	$-.80188 \cdot 10^{-4}$
7.50	.00011401	$-.17958 \cdot 10^{-4}$	$-.71977 \cdot 10^{-4}$
8.00	$.58866 \cdot 10^{-4}$	$-.36510 \cdot 10^{-4}$	$-.52641 \cdot 10^{-4}$
8.50	$.23265 \cdot 10^{-4}$	$-.38614 \cdot 10^{-4}$	$-.32759 \cdot 10^{-4}$
9.00	$+.26115 \cdot 10^{-5}$	$-.32928 \cdot 10^{-4}$	$-.16878 \cdot 10^{-4}$
9.50	$-.78109 \cdot 10^{-5}$	$-.24735 \cdot 10^{-4}$	$-.60498 \cdot 10^{-5}$
10.00	$-.11852 \cdot 10^{-4}$	$-.16817 \cdot 10^{-4}$	$+.03393 \cdot 10^{-6}$
20.00			
30.00	$+.90 \cdot 10^{-9}$	$-.31 \cdot 10^{-9}$	
40.00	$.60894 \cdot 10^{-10}$	$-.110 \cdot 10^{-10}$	$.3 \cdot 10^{-11}$
50.00	$.88676 \cdot 10^{-11}$	$-.11215 \cdot 10^{-11}$	$.1872 \cdot 10^{-12}$
60.00	$.19399 \cdot 10^{-11}$	$-.19009 \cdot 10^{-12}$	$.24038 \cdot 10^{-13}$
70.00	$.55100 \cdot 10^{-12}$	$-.44174 \cdot 10^{-13}$	$.45160 \cdot 10^{-14}$
80.00	$.18798 \cdot 10^{-12}$	$-.12764 \cdot 10^{-13}$	$.10970 \cdot 10^{-14}$
90.00	$.73482 \cdot 10^{-13}$	$-.43298 \cdot 10^{-14}$	$.32120 \cdot 10^{-15}$
100.00	$.31914 \cdot 10^{-13}$	$-.16613 \cdot 10^{-14}$	$.10846 \cdot 10^{-15}$

Table 5. $\alpha = -1/2 \longrightarrow 19/2;\ \beta = 5.$

p	${}_1F_1(-\frac{1}{2};5;-p)$	${}_1F_1(\frac{1}{2};5;-p)$	${}_1F_1(\frac{3}{2};5;-p)$	${}_1F_1(\frac{5}{2};5;-p)$
0	1.0	1.0	1.0	1.0
.25	1.02474	.97576	.92875	.88365
.50	1.04899	.95295	.86441	.78286
.75	1.07277	.93145	.80618	.69538
1.00	1.09611	.91116	.75339	.61926
1.25	1.11902	.89198	.70541	.55290
1.50	1.14152	.87382	.66172	.49490
1.75	1.16364	.85660	.62185	.44410
2.00	1.18539	.84026	.58541	.39951
2.50	1.22785	.80994	.52139	.32568
3.00	1.26901	.78242	.46728	.26802
3.50	1.30898	.75731	.42122	.22259
4.00	1.34785	.73431	.38173	.18648
4.50	1.38570	.71315	.34765	.15752
5.00	1.42260	.69363	.31808	.13412
5.50	1.45862	.67554	.29224	.11504
6.00	1.49380	.65873	.26956	.099362
6.50	1.52822	.64307	.24955	.086388
7.00	1.56191	.62843	.23179	.075570
7.50	1.59491	.61471	.21597	.066488
8.00	1.62727	.60182	.20181	.058813
8.50	1.65901	.58968	.18909	.052286
9.00	1.69018	.57823	.17761	.046703
9.50	1.72080	.56741	.16722	.041899
10.00	1.75089	.55715	.15778	.037745
20.00	2.27174	.42456	.066978	.0082865
30.00	2.69448	.35605	.038792	.0032184
40.00	3.05957	.31262	.025993	.0016206
50.00	3.38560	.28196	.018951	.00094606
60.00	3.68293	.25884	.014597	.00060757
70.00	3.95801	.24061	.011688	.00041709
80.00	4.21519	.22576	.0096304	.00030077
90.00	4.45757	.21335	.0081129	.00022525
100.00	4.68743	.20279	.0069559	.00017383

Table 5. (Cont.) $\alpha = -1/2 \longrightarrow 19/2$; $\beta = 5$.

p	${}_1F_1(\frac{7}{2};5;-p)$	${}_1F_1(\frac{9}{2};5;-p)$	${}_1F_1(\frac{11}{2};5;-p)$	${}_1F_1(\frac{13}{2};5;-p)$
0	1.0	1.0	1.0	1.0
.25	.84038	.79890	.75912	.72100
.50	.70784	.63887	.57555	.51747
.75	.59757	.51144	.43577	.36947
1.00	.50568	.40988	.32944	.26223
1.25	.42896	.32888	.24864	.18484
1.50	.36478	.26421	.18731	.12924
1.75	.31098	.21254	.14082	.089497
2.00	.26580	.17122	.10563	.061257
2.50	.19572	.11162	.058952	.027368
3.00	.14566	.073250	.032462	.011047
3.50	.10959	.048429	.017558	.0035781
4.00	.083362	.032284	.0092624	+.00043324
4.50	.064109	.021718	.0047101	-.00068982
5.00	.049842	.014757	.0022585	-.00093093
5.50	.039164	.010137	.00097264	-.00083312
6.00	.031096	.0070450	.00032402	-.00064046
6.50	.024940	.0049575	$+.16922 \cdot 10^{-4}$	-.00045222
7.00	.020197	.0035345	-.00011222	-.00030091
7.50	.016508	.0025543	-.00015248	-.00019062
8.00	.013612	.0018716	-.00015129	-.00011513
8.50	.011317	.0013907	-.00013322	$-.65872 \cdot 10^{-4}$
9.00	.0094839	.0010477	-.00011038	$-.35041 \cdot 10^{-4}$
9.50	.0080065	.00080012	$-.88313 \cdot 10^{-4}$	$-.16538 \cdot 10^{-4}$
10.00	.0068063	.00061908	$-.69193 \cdot 10^{-4}$	$-.59578 \cdot 10^{-5}$
20.00	.00068632	$.2168 \cdot 10^{-4}$	$-.822 \cdot 10^{-6}$	$+.1 \cdot 10^{-6}$
30.00	.00017200	$.33198 \cdot 10^{-5}$	$-.70308 \cdot 10^{-7}$	$.4996 \cdot 10^{-8}$
40.00	$.63889 \cdot 10^{-4}$	$.88899 \cdot 10^{-6}$	$-.13175 \cdot 10^{-7}$	$.62859 \cdot 10^{-9}$
50.00	$.29541 \cdot 10^{-4}$	$.32145 \cdot 10^{-6}$	$-.36694 \cdot 10^{-8}$	$.13235 \cdot 10^{-9}$
60.00	$.15704 \cdot 10^{-4}$	$.14032 \cdot 10^{-6}$	$-.13031 \cdot 10^{-8}$	$.37830 \cdot 10^{-10}$
70.00	$.91969 \cdot 10^{-5}$	$.69711 \cdot 10^{-7}$	$-.54575 \cdot 10^{-9}$	$.13262 \cdot 10^{-10}$
80.00	$.57823 \cdot 10^{-5}$	$.38057 \cdot 10^{-7}$	$-.25754 \cdot 10^{-9}$	$.53829 \cdot 10^{-11}$
90.00	$.38386 \cdot 10^{-5}$	$.22325 \cdot 10^{-7}$	$-.13304 \cdot 10^{-9}$	$.24398 \cdot 10^{-11}$
100.00	$.26602 \cdot 10^{-5}$	$.13858 \cdot 10^{-7}$	$-.73786 \cdot 10^{-10}$	$.12054 \cdot 10^{-11}$

Table 5. (Cont.) $\alpha = 1/2 \longrightarrow 19/2;\ \beta = 5.$

p	${}_1F_1(\frac{15}{2};5;-p)$	${}_1F_1(\frac{17}{2};5;-p)$	${}_1F_1(\frac{19}{2};5;-p)$
0	1.0	1.0	1.0
.25	.68447	.64948	.61597
.50	.46426	.41557	.37108
.75	.31154	.26107	.21726
1.00	.20638	.16024	.12239
1.25	.13457	.095384	.065222
1.50	.086012	.054401	.031784
1.75	.053557	.029080	.013014
2.00	.032169	.013894	+.0031002
2.50	.0095531	+.00043048	-.0034525
3.00	+.0010068	-.0027428	-.0033187
3.50	-.0015746	-.0025574	-.0019090
4.00	-.0018709	-.0016411	-.00077421
4.50	-.0014584	-.00083955	-.00014026
5.00	-.00095086	-.00032360	+.00012504
5.50	-.00054488	$-.49226 \cdot 10^{-4}$	.00018672
6.00	-.00027184	$+.68506 \cdot 10^{-4}$	.00016029
6.50	-.00010826	.00010022	.00010943
7.00	$-.20396 \cdot 10^{-4}$	$.92146 \cdot 10^{-4}$	$.62602 \cdot 10^{-4}$
7.50	$+.20523 \cdot 10^{-4}$	$.70382 \cdot 10^{-4}$	$.28810 \cdot 10^{-4}$
8.00	$.34914 \cdot 10^{-4}$	$.47688 \cdot 10^{-4}$	$+.80653 \cdot 10^{-5}$
8.50	$.35810 \cdot 10^{-4}$	$.29119 \cdot 10^{-4}$	$-.27550 \cdot 10^{-5}$
9.00	$.30863 \cdot 10^{-4}$	$.15796 \cdot 10^{-4}$	$-.71334 \cdot 10^{-5}$
9.50	$.24196 \cdot 10^{-4}$	$.71259 \cdot 10^{-5}$	$-.78674 \cdot 10^{-5}$
10.00	$.17801 \cdot 10^{-4}$	$.19859 \cdot 10^{-5}$	$-.68625 \cdot 10^{-5}$
20.00			
30.00	$-.68 \cdot 10^{-9}$		
40.00	$-.54180 \cdot 10^{-10}$	$.7193 \cdot 10^{-11}$	$-.138 \cdot 10^{-11}$
50.00	$-.84220 \cdot 10^{-11}$	$.79913 \cdot 10^{-12}$	$-.1047 \cdot 10^{-12}$
60.00	$-.19128 \cdot 10^{-11}$	$.14200 \cdot 10^{-12}$	$-.14275 \cdot 10^{-13}$
70.00	$-.55684 \cdot 10^{-12}$	$.34010 \cdot 10^{-13}$	$-.27823 \cdot 10^{-14}$
80.00	$-.19332 \cdot 10^{-12}$	$.10037 \cdot 10^{-13}$	$-.69307 \cdot 10^{-15}$
90.00	$-.76568 \cdot 10^{-13}$	$.34583 \cdot 10^{-14}$	$-.20671 \cdot 10^{-15}$
100.00	$-.33595 \cdot 10^{-13}$	$.13430 \cdot 10^{-14}$	$-.70791 \cdot 10^{-16}$

Table 6. $\alpha = -1/2 \longrightarrow 21/2$; $\beta = 6$.

p	${}_1F_1(-\frac{1}{2};6;-p)$	${}_1F_1(\frac{1}{2};6;-p)$	${}_1F_1(\frac{3}{2};6;-p)$	${}_1F_1(\frac{5}{2};6;-p)$
0	1.0	1.0	1.0	1.0
.25	1.02065	.97971	.94019	.90205
.50	1.04094	.96045	.88540	.81544
.75	1.06090	.94216	.83510	.73871
1.00	1.08053	.92475	.78886	.67060
1.25	1.09985	.90816	.74628	.61003
1.50	1.11887	.89235	.70700	.55605
1.75	1.13761	.87726	.67071	.50786
2.00	1.15607	.86283	.63712	.46475
2.50	1.19221	.83581	.57709	.39143
3.00	1.22737	.81099	.52522	.33210
3.50	1.26163	.78810	.48013	.28375
4.00	1.29504	.76693	.44073	.24406
4.50	1.32766	.74727	.40611	.21125
5.00	1.35954	.72897	.37555	.18396
5.50	1.39073	.71188	.34845	.16110
6.00	1.42127	.69589	.32431	.14184
6.50	1.45119	.68089	.30271	.12551
7.00	1.48053	.66677	.28331	.11159
7.50	1.50933	.65347	.26582	.099656
8.00	1.53760	.64091	.25000	.089375
8.50	1.56538	.62902	.23564	.080473
9.00	1.59269	.61775	.22256	.072728
9.50	1.61955	.60705	.21062	.065960
10.00	1.64598	.59687	.19968	.060020
20.00	2.10720	.46180	.089394	.014673
30.00	2.48496	.38974	.052876	.0059290
40.00	2.81264	.34337	.035828	.0030466
50.00	3.10604	.31036	.026301	.0018005
60.00	3.37406	.28534	.020354	.0011658
70.00	3.62233	.26553	.016352	.00080504
80.00	3.85466	.24934	.013508	.00058310
90.00	4.07377	.23579	.011402	.00043820
100.00	4.28169	.22423	.0097916	.00033910

Table 6. (Cont.) $\alpha = -1/2 \longrightarrow 21/2$; $\beta = 6$.

p	${}_1F_1(\frac{7}{2};6;-p)$	${}_1F_1(\frac{9}{2};6;-p)$	${}_1F_1(\frac{11}{2};6;-p)$	${}_1F_1(\frac{13}{2};6;-p)$
0	1.0	1.0	1.0	1.0
.25	.86524	.82973	.79547	.76243
.50	.75029	.68964	.63323	.58079
.75	.65204	.57423	.50446	.44201
1.00	.56793	.47900	.40220	.33605
1.25	.49577	.40032	.32094	.25521
1.50	.43375	.33522	.25632	.19359
1.75	.38034	.28126	.20491	.14665
2.00	.33427	.23646	.16397	.11093
2.50	.25993	.16819	.10533	.063168
3.00	.20394	.12068	.067980	.035691
3.50	.16143	.087372	.044102	.019971
4.00	.12890	.063847	.028777	.011036
4.50	.10380	.047101	.018898	.0059999
5.00	.084275	.035084	.012499	.0031895
5.50	.068975	.026389	.0083312	.0016416
6.00	.056888	.020043	.0056009	.00080373
6.50	.047267	.015371	.0038005	.00036088
7.00	.039552	.011902	.0026048	.00013478
7.50	.033320	.0093023	.0018045	$+.25432\cdot10^{-4}$
8.00	.028251	.0073374	.0012643	$-.22599\cdot10^{-4}$
8.50	.024099	.0058392	.00089642	$-.39616\cdot10^{-4}$
9.00	.020677	.0046868	.00064339	$-.41854\cdot10^{-4}$
9.50	.017838	.0037928	.00046759	$-.37776\cdot10^{-4}$
10.00	.015469	.0030936	.00034414	$-.31618\cdot10^{-4}$
20.00	.0019000	.00016616	$.5626\cdot10^{-5}$	$-.24\cdot10^{-6}$
30.00	.00050773	$.28113\cdot10^{-4}$	$.56502\cdot10^{-6}$	$-.12550\cdot10^{-7}$
40.00	.00019459	$.78750\cdot10^{-5}$	$.11277\cdot10^{-6}$	$-.17255\cdot10^{-8}$
50.00	$.91652\cdot10^{-4}$	$.29219\cdot10^{-5}$	$.32512\cdot10^{-7}$	$-.38018\cdot10^{-9}$
60.00	$.49322\cdot10^{-4}$	$.12970\cdot10^{-5}$	$.11802\cdot10^{-7}$	$-.11175\cdot10^{-9}$
70.00	$.29135\cdot10^{-4}$	$.65194\cdot10^{-6}$	$.50183\cdot10^{-8}$	$-.39930\cdot10^{-10}$
80.00	$.18437\cdot10^{-4}$	$.35902\cdot10^{-6}$	$.23946\cdot10^{-8}$	$-.16433\cdot10^{-10}$
90.00	$.12301\cdot10^{-4}$	$.21202\cdot10^{-6}$	$.12476\cdot10^{-8}$	$-.75269\cdot10^{-11}$
100.00	$.85585\cdot10^{-5}$	$.13232\cdot10^{-6}$	$.69661\cdot10^{-9}$	$-.37496\cdot10^{-11}$

Table 6. (Cont.) $\alpha = -1/2 \rightarrow 21/2;\ \beta = 6.$

p	${}_1F_1(\frac{15}{2};6;-p)$	${}_1F_1(\frac{17}{2};6;-p)$	${}_1F_1(\frac{19}{2};6;-p)$	${}_1F_1(\frac{21}{2};6;-p)$
0	1.0	1.0	1.0	1.0
.25	.73056	.69984	.67022	.64167
.50	.53208	.48687	.44493	.40606
.75	.38621	.33643	.29210	.25271
1.00	.27926	.23067	.18924	.15406
1.25	.20108	.15674	.12065	.091476
1.50	.14409	.10537	.075388	.052439
1.75	.10269	.069934	.045902	.028593
2.00	.072721	.045686	.026985	.014414
2.50	.035630	.018245	.0077660	+.0018615
3.00	.016734	.0062493	+.00095985	-.0012920
3.50	.0073610	+.0014039	-.00092624	-.0014435
4.00	.0028801	-.00028720	-.0010836	-.00092077
4.50	.00085397	-.00068760	-.00077698	-.00044187
5.00	$+.19934 \cdot 10^{-4}$	-.00062726	-.00044864	-.00014670
5.50	-.00026203	-.00045060	-.00021450	$-.33296 \cdot 10^{-5}$
6.00	-.00030718	-.00028362	$-.76487 \cdot 10^{-4}$	$+.48133 \cdot 10^{-4}$
6.50	-.00026458	-.00016037	$-.70834 \cdot 10^{-5}$	$.54238 \cdot 10^{-4}$
7.00	-.00020037	$-.80387 \cdot 10^{-4}$	$+.21103 \cdot 10^{-4}$	$.42944 \cdot 10^{-4}$
7.50	-.00014076	$-.33239 \cdot 10^{-4}$	$.27714 \cdot 10^{-4}$	$.28291 \cdot 10^{-4}$
8.00	$-.93780 \cdot 10^{-4}$	$-.79841 \cdot 10^{-5}$	$.24764 \cdot 10^{-4}$	$.15975 \cdot 10^{-4}$
8.50	$-.59813 \cdot 10^{-4}$	$+.39357 \cdot 10^{-5}$	$.18750 \cdot 10^{-4}$	$.74314 \cdot 10^{-5}$
9.00	$-.36614 \cdot 10^{-4}$	$.83709 \cdot 10^{-5}$	$.12738 \cdot 10^{-4}$	$+.22795 \cdot 10^{-5}$
9.50	$-.21440 \cdot 10^{-4}$	$.89845 \cdot 10^{-5}$	$.78912 \cdot 10^{-5}$	$-.04028 \cdot 10^{-5}$
10.00	$-.11879 \cdot 10^{-4}$	$+.79075 \cdot 10^{-5}$	$.44242 \cdot 10^{-5}$	$-.15162 \cdot 10^{-5}$
20.00	$+.4 \cdot 10^{-7}$			
30.00	$.947 \cdot 10^{-9}$	$-.141 \cdot 10^{-9}$	$.4 \cdot 10^{-10}$	
40.00	$.85346 \cdot 10^{-10}$	$-.7672 \cdot 10^{-11}$	$.107 \cdot 10^{-11}$	$-.22 \cdot 10^{-12}$
50.00	$.14078 \cdot 10^{-10}$	$-.92212 \cdot 10^{-12}$	$.90383 \cdot 10^{-13}$	$-.1229 \cdot 10^{-13}$
60.00	$.33119 \cdot 10^{-11}$	$-.17123 \cdot 10^{-12}$	$.13023 \cdot 10^{-13}$	$-.13443 \cdot 10^{-14}$
70.00	$.98707 \cdot 10^{-12}$	$-.42204 \cdot 10^{-13}$	$.26280 \cdot 10^{-14}$	$-.21950 \cdot 10^{-15}$
80.00	$.34851 \cdot 10^{-12}$	$-.12710 \cdot 10^{-13}$	$.67063 \cdot 10^{-15}$	$-.47106 \cdot 10^{-16}$
90.00	$.13980 \cdot 10^{-12}$	$-.44459 \cdot 10^{-14}$	$.20361 \cdot 10^{-15}$	$-.12349 \cdot 10^{-16}$
100.00	$.61950 \cdot 10^{-13}$	$-.17469 \cdot 10^{-14}$	$.70690 \cdot 10^{-16}$	$-.37737 \cdot 10^{-17}$

Table 7. $\alpha = -1/2 \longrightarrow 21/2;\ \ \beta = 7.$

p	${}_1F_1(-\frac{1}{2};7;-p)$	${}_1F_1(\frac{1}{2};7;-p)$	${}_1F_1(\frac{3}{2};7;-p)$	${}_1F_1(\frac{5}{2};7;-p)$
0	1.0	1.0	1.0	1.0
.25	1.01772	.98255	.94846	.91540
.50	1.03517	.96588	.90071	.83944
.75	1.05236	.94995	.85643	.77111
1.00	1.06931	.93470	.81530	.70956
1.25	1.08602	.92009	.77704	.65401
1.50	1.10250	.90608	.74140	.60380
1.75	1.11876	.89263	.70817	.55834
2.00	1.13481	.87971	.67713	.51711
2.50	1.16630	.85535	.62093	.44559
3.00	1.19702	.83276	.57155	.38623
3.50	1.22702	.81175	.52796	.33665
4.00	1.25636	.79217	.48930	.29500
4.50	1.28506	.77385	.45488	.25981
5.00	1.31317	.75668	.42410	.22991
5.50	1.34072	.74056	.39647	.20438
6.00	1.36774	.72538	.37158	.18247
6.50	1.39426	.71105	.34909	.16357
7.00	1.42030	.69751	.32868	.14719
7.50	1.44589	.68468	.31012	.13293
8.00	1.47105	.67252	.29318	.12047
8.50	1.49581	.66096	.27768	.10953
9.00	1.52017	.64996	.26346	.099891
9.50	1.54416	.63947	.25037	.091366
10.00	1.56779	.62947	.23831	.083798
20.00	1.98308	.49362	.11172	.022416
30.00	2.32604	.41904	.067372	.0093895
40.00	2.62478	.37039	.046131	.0049172
50.00	2.89292	.33548	.034088	.0029400
60.00	3.13828	.30887	.026499	.0019188
70.00	3.36582	.28773	.021358	.0013326
80.00	3.57895	.27040	.017687	.00096936
90.00	3.78008	.25586	.014959	.00073093
100.00	3.97105	.24345	.012866	.00056715

Table 7. (Cont.) $\alpha = -1/2 \longrightarrow 21/2$; $\beta = 7$.

p	${}_1F_1(\frac{7}{2};7;-p)$	${}_1F_1(\frac{9}{2};7;-p)$	${}_1F_1(\frac{11}{2};7;-p)$	${}_1F_1(\frac{13}{2};7;-p)$
0	1.0	1.0	1.0	1.0
.25	.88336	.85230	.82221	.79304
.50	.78185	.72774	.67694	.62926
.75	.69335	.62254	.55812	.49958
1.00	.61607	.53354	.46082	.39687
1.25	.54846	.45813	.38105	.31547
1.50	.48921	.39414	.31558	.25094
1.75	.43720	.33974	.26176	.19974
2.00	.39145	.29342	.21747	.15910
2.50	.31560	.22016	.15087	.10119
3.00	.25633	.16652	.10540	.064578
3.50	.20969	.12696	.074178	.041368
4.00	.17275	.097573	.052605	.026611
4.50	.14328	.075592	.037604	.017197
5.00	.11962	.059029	.027102	.011171
5.50	.10050	.046457	.019699	.0072977
6.00	.084948	.036845	.014442	.0047971
6.50	.072222	.029443	.010681	.0031750
7.00	.061745	.023700	.0079688	.0021172
7.50	.053069	.019214	.0059982	.0014233
8.00	.045843	.015685	.0045548	.00096520
8.50	.039793	.012890	.0034890	.00066073
9.00	.034701	.010660	.0026956	.00045683
9.50	.030393	.0088708	.0021001	.00031918
10.00	.026730	.0074253	.0016497	.00022545
20.00	.0038318	.00052016	$.4816 \cdot 10^{-4}$	$.176 \cdot 10^{-5}$
30.00	.0010842	$.95923 \cdot 10^{-4}$	$.55096 \cdot 10^{-5}$	$.11551 \cdot 10^{-6}$
40.00	.00042780	$.28007 \cdot 10^{-4}$	$.11643 \cdot 10^{-5}$	$.17174 \cdot 10^{-7}$
50.00	.00020506	$.10648 \cdot 10^{-4}$	$.34673 \cdot 10^{-6}$	$.39471 \cdot 10^{-8}$
60.00	.00011165	$.48025 \cdot 10^{-5}$	$.12852 \cdot 10^{-6}$	$.11914 \cdot 10^{-8}$
70.00	$.66506 \cdot 10^{-4}$	$.24414 \cdot 10^{-5}$	$.55450 \cdot 10^{-7}$	$.43356 \cdot 10^{-9}$
80.00	$.42350 \cdot 10^{-4}$	$.13558 \cdot 10^{-5}$	$.26747 \cdot 10^{-7}$	$.18083 \cdot 10^{-9}$
90.00	$.28394 \cdot 10^{-4}$	$.80592 \cdot 10^{-6}$	$.14051 \cdot 10^{-7}$	$.83679 \cdot 10^{-10}$
100.00	$.19832 \cdot 10^{-4}$	$.50557 \cdot 10^{-6}$	$.78971 \cdot 10^{-8}$	$.42022 \cdot 10^{-10}$

Table 7. (Cont.) $\alpha = -1/2 \rightarrow 21/2$; $\beta = 7$.

p	${}_1F_1(\frac{15}{2};7;-p)$	${}_1F_1(\frac{17}{2};7;-p)$	${}_1F_1(\frac{19}{2};7;-p)$	${}_1F_1(\frac{21}{2};7;-p)$
0	1.0	1.0	1.0	1.0
.25	.76478	.73741	.71089	.68520
.50	.58452	.54257	.50325	.46642
.75	.44644	.39826	.35461	.31513
1.00	.34073	.29156	.24858	.21110
1.25	.25985	.21283	.17324	.14002
1.50	.19800	.15487	.11993	.091798
1.75	.15073	.11230	.082393	.059346
2.00	.11464	.081105	.056103	.037713
2.50	.066093	.041722	.025150	.014171
3.00	.037913	.020970	.010579	.0045037
3.50	.021617	.010212	.0039946	+.00088669
4.00	.012234	.0047510	.0011946	-.00024426
4.50	.0068613	.0020554	+.00011917	-.00044682
5.00	.0038034	.00077664	-.00021435	-.00036232
5.50	.0020767	+.00020571	-.00025757	-.00023036
6.00	.0011109	$-.23564 \cdot 10^{-4}$	-.00020713	-.00012462
6.50	.00057735	$-.96197 \cdot 10^{-4}$	-.00014150	$-.56604 \cdot 10^{-4}$
7.00	.00028727	-.00010284	$-.86991 \cdot 10^{-4}$	$-.18721 \cdot 10^{-4}$
7.50	.00013296	$-.86020 \cdot 10^{-4}$	$-.48763 \cdot 10^{-4}$	$-.04614 \cdot 10^{-5}$
8.00	$.53385 \cdot 10^{-4}$	$-.64347 \cdot 10^{-4}$	$-.24561 \cdot 10^{-4}$	$+.65918 \cdot 10^{-5}$
8.50	$+.14257 \cdot 10^{-4}$	$-.44999 \cdot 10^{-4}$	$-.10457 \cdot 10^{-4}$	$.79893 \cdot 10^{-5}$
9.00	$-.34939 \cdot 10^{-5}$	$-.29990 \cdot 10^{-4}$	$-.29116 \cdot 10^{-5}$	$.69726 \cdot 10^{-5}$
9.50	$-.10318 \cdot 10^{-4}$	$-.19215 \cdot 10^{-4}$	$+.06905 \cdot 10^{-5}$	$.52383 \cdot 10^{-5}$
10.00	$-.11843 \cdot 10^{-4}$	$-.11872 \cdot 10^{-4}$	$+.20899 \cdot 10^{-5}$	$.35642 \cdot 10^{-5}$
20.00	$-.83 \cdot 10^{-7}$			
30.00	$-.26995 \cdot 10^{-8}$	$+218 \cdot 10^{-9}$	$-.4 \cdot 10^{-10}$	$.1 \cdot 10^{-10}$
40.00	$-.27162 \cdot 10^{-9}$	$.13953 \cdot 10^{-10}$	$-.1311 \cdot 10^{-11}$	$.194 \cdot 10^{-12}$
50.00	$-.47311 \cdot 10^{-10}$	$.18000 \cdot 10^{-11}$	$-.12150 \cdot 10^{-12}$	$.12321 \cdot 10^{-13}$
60.00	$-.11506 \cdot 10^{-10}$	$.34831 \cdot 10^{-12}$	$-.18426 \cdot 10^{-13}$	$.14367 \cdot 10^{-14}$
70.00	$-.35072 \cdot 10^{-11}$	$.88223 \cdot 10^{-13}$	$-.38427 \cdot 10^{-14}$	$.24407 \cdot 10^{-15}$
80.00	$-.12586 \cdot 10^{-11}$	$.27092 \cdot 10^{-13}$	$-.10035 \cdot 10^{-14}$	$.53830 \cdot 10^{-16}$
90.00	$-.51112 \cdot 10^{-12}$	$.96163 \cdot 10^{-14}$	$-.30997 \cdot 10^{-15}$	$.14397 \cdot 10^{-16}$
100.00	$-.22869 \cdot 10^{-12}$	$.38218 \cdot 10^{-14}$	$-.10906 \cdot 10^{-15}$	$.44678 \cdot 10^{-17}$

Table 8. $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 8$.

p	${}_1F_1(-\frac{1}{2};8;-p)$	${}_1F_1(\frac{1}{2};8;-p)$	${}_1F_1(\frac{3}{2};8;-p)$	${}_1F_1(\frac{5}{2};8;-p)$	${}_1F_1(\frac{7}{2};8;-p)$
0	1.0	1.0	1.0	1.0	1.0
.25	1.01552	.98469	.95471	.92553	.89716
.50	1.03083	.97000	.91240	.85787	.80626
.75	1.04593	.95588	.87283	.79629	.72580
1.00	1.06085	.94231	.83579	.74018	.65445
1.25	1.07557	.92924	.80106	.68896	.59110
1.50	1.09011	.91666	.76847	.64216	.53475
1.75	1.10448	.90454	.73785	.59932	.48456
2.00	1.11868	.89284	.70905	.56006	.43979
2.50	1.14659	.87065	.65638	.49093	.36398
3.00	1.17388	.84993	.60949	.43241	.30310
3.50	1.20059	.83054	.56760	.38261	.25391
4.00	1.22675	.81233	.53001	.34003	.21394
4.50	1.25240	.79521	.49618	.30344	.18127
5.00	1.27756	.77908	.46562	.27186	.15441
5.50	1.30226	.76384	.43793	.24448	.13222
6.00	1.32652	.74942	.41275	.22063	.11378
6.50	1.35036	.73576	.38980	.19979	.098374
7.00	1.37380	.72279	.36882	.18149	.085445
7.50	1.39686	.71046	.34959	.16537	.074540
8.00	1.41956	.69872	.33192	.15112	.065298
8.50	1.44192	.68752	.31564	.13847	.057431
9.00	1.46395	.67683	.30061	.12722	.050704
9.50	1.48566	.66661	.28670	.11716	.044927
10.00	1.50706	.65682	.27381	.10816	.039947
20.00	1.88563	.52131	.13366	.031256	.0065046
30.00	2.20064	.44496	.082057	.013529	.0019379
40.00	2.47609	.39452	.056745	.0072125	.00078564
50.00	2.72393	.35804	.042195	.0043607	.00038290
60.00	2.95106	.33010	.032944	.0028676	.00021083
70.00	3.16196	.30781	.026637	.0020025	.00012661
80.00	3.35965	.28950	.022112	.0014628	$.81113 \cdot 10^{-4}$
90.00	3.54635	.27410	.018737	.0011066	$.54642 \cdot 10^{-4}$
100.00	3.72371	.26093	.016141	.00086095	$.38312 \cdot 10^{-4}$

Table 8. (Cont.) $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 8$.

p	${}_1F_1(\frac{9}{2};8;-p)$	${}_1F_1(\frac{11}{2};8;-p)$	${}_1F_1(\frac{13}{2};8;-p)$	${}_1F_1(\frac{15}{2};8;-p)$
0	1.0	1.0	1.0	1.0
.25	.86956	.84272	.81661	.79123
.50	.75743	.71125	.66758	.62631
.75	.66091	.60122	.54637	.49598
1.00	.57768	.50902	.44768	.39296
1.25	.50582	.43164	.36726	.31149
1.50	.44366	.36662	.30165	.24703
1.75	.38983	.31191	.24808	.19602
2.00	.34312	.26581	.20429	.15563
2.50	.26722	.19402	.13910	.098275
3.00	.20956	.14261	.095254	.062218
3.50	.16547	.10556	.065619	.039502
4.00	.13156	.078694	.045490	.025158
4.50	.10529	.059092	.031744	.016078
5.00	.084827	.044697	.022304	.010315
5.50	.068780	.034056	.015784	.0066449
6.00	.056120	.026137	.011252	.0043006
6.50	.046070	.020205	.0080830	.0027975
7.00	.038044	.015732	.0058516	.0018299
7.50	.031597	.012335	.0042700	.0012043
8.00	.026388	.0097391	.0031409	.00079784
8.50	.022156	.0077417	.0023292	.00053239
9.00	.018698	.0061948	.0017412	.00035803
9.50	.015858	.0049889	.0013123	.00024279
10.00	.013514	.0040430	.00099695	.00016611
20.00	.0011591	.00016520	$.1624 \cdot 10^{-4}$	$.64 \cdot 10^{-6}$
30.00	.00023061	$.21096 \cdot 10^{-4}$	$.12586 \cdot 10^{-5}$	$.27583 \cdot 10^{-7}$
40.00	$.69964 \cdot 10^{-4}$	$.46975 \cdot 10^{-5}$	$.20075 \cdot 10^{-6}$	$.30531 \cdot 10^{-8}$
50.00	$.27217 \cdot 10^{-4}$	$.14421 \cdot 10^{-5}$	$.47990 \cdot 10^{-7}$	$.55922 \cdot 10^{-9}$
60.00	$.12465 \cdot 10^{-4}$	$.54530 \cdot 10^{-6}$	$.14855 \cdot 10^{-7}$	$.14034 \cdot 10^{-9}$
70.00	$.64065 \cdot 10^{-5}$	$.23860 \cdot 10^{-6}$	$.55017 \cdot 10^{-8}$	$.43707 \cdot 10^{-10}$
80.00	$.35870 \cdot 10^{-5}$	$.11629 \cdot 10^{-6}$	$.23245 \cdot 10^{-8}$	$.15933 \cdot 10^{-10}$
90.00	$.21457 \cdot 10^{-5}$	$.61590 \cdot 10^{-7}$	$.10864 \cdot 10^{-8}$	$.65481 \cdot 10^{-11}$
100.00	$.13529 \cdot 10^{-5}$	$.34837 \cdot 10^{-7}$	$.54986 \cdot 10^{-9}$	$.29575 \cdot 10^{-11}$

Table 8. (Cont.) $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 8$.

p	${}_1F_1(\frac{17}{2};8;-p)$	${}_1F_1(\frac{19}{2};8;-p)$	${}_1F_1(\frac{21}{2};8;-p)$	${}_1F_1(\frac{23}{2};8;-p)$
0	1.0	1.0	1.0	1.0
.25	.76655	.74255	.71922	.69654
.50	.58731	.55046	.51568	.48284
.75	.44974	.40734	.36849	.33292
1.00	.34421	.30085	.26234	.22818
1.25	.26329	.22174	.18600	.15535
1.50	.20127	.16306	.13128	.10496
1.75	.15375	.11961	.092188	.070293
2.00	.11737	.087505	.064367	.046598
2.50	.068239	.046402	.030743	.019695
3.00	.039533	.024246	.014176	.0077277
3.50	.022809	.012435	.0062158	.0026630
4.00	.013096	.0062237	.0025180	+.00067651
4.50	.0074758	.0030120	.00088043	$-.44038\cdot10^{-5}$
5.00	.0042375	.0013874	+.00020716	-.00017250
5.50	.0023812	.00058963	$-.34620\cdot10^{-4}$	-.00016512
6.00	.0013236	.00021416	$-.96266\cdot10^{-4}$	-.00011517
6.50	.00072536	$+.48784\cdot10^{-4}$	$-.91423\cdot10^{-4}$	$-.68210\cdot10^{-4}$
7.00	.00039011	$-.15850\cdot10^{-4}$	$-.68270\cdot10^{-4}$	$-.35238\cdot10^{-4}$
7.50	.00020438	$-.34773\cdot10^{-4}$	$-.45082\cdot10^{-4}$	$-.15335\cdot10^{-4}$
8.00	.00010302	$-.34812\cdot10^{-4}$	$-.27259\cdot10^{-4}$	$-.46918\cdot10^{-5}$
8.50	$.48799\cdot10^{-4}$	$-.28446\cdot10^{-4}$	$-.15191\cdot10^{-4}$	$+.02626\cdot10^{-5}$
9.00	$.20608\cdot10^{-4}$	$-.21061\cdot10^{-4}$	$-.76877\cdot10^{-5}$	$.20858\cdot10^{-5}$
9.50	$.65558\cdot10^{-5}$	$-.14667\cdot10^{-4}$	$-.33510\cdot10^{-5}$	$.23752\cdot10^{-5}$
10.00	$+.00203\cdot10^{-5}$	$-.97734\cdot10^{-5}$	$-.10320\cdot10^{-5}$	$.20321\cdot10^{-5}$
20.00	$-.3\cdot10^{-7}$			
30.00	$-.6806\cdot10^{-9}$	$.591\cdot10^{-10}$	$-.1\cdot10^{-10}$	
40.00	$-.49975\cdot10^{-10}$	$.26712\cdot10^{-11}$	$-.263\cdot10^{-12}$	$.42\cdot10^{-13}$
50.00	$-.68755\cdot10^{-11}$	$.26901\cdot10^{-12}$	$-.18735\cdot10^{-13}$	$.19690\cdot10^{-14}$
60.00	$-.13830\cdot10^{-11}$	$.42786\cdot10^{-13}$	$-.23173\cdot10^{-14}$	$.18540\cdot10^{-15}$
70.00	$-.35954\cdot10^{-12}$	$.92066\cdot10^{-14}$	$-.40868\cdot10^{-15}$	$.26490\cdot10^{-16}$
80.00	$-.11250\cdot10^{-12}$	$.24583\cdot10^{-14}$	$-.92520\cdot10^{-16}$	$.50468\cdot10^{-17}$
90.00	$-.40501\cdot10^{-13}$	$.77205\cdot10^{-15}$	$-.25228\cdot10^{-16}$	$.11887\cdot10^{-17}$
100.00	$-.16276\cdot10^{-13}$	$.27516\cdot10^{-15}$	$-.79467\cdot10^{-17}$	$.32966\cdot10^{-18}$

Table 9. $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 9$.

p	${}_1F_1(-\frac{1}{2};9;-p)$	${}_1F_1(\frac{1}{2};9;-p)$	${}_1F_1(\frac{3}{2};9;-p)$	${}_1F_1(\frac{5}{2};9;-p)$	${}_1F_1(\frac{7}{2};9;-p)$
0	1.0	1.0	1.0	1.0	1.0
.25	1.01380	.98637	.95960	.93349	.90803
.50	1.02744	.97322	.92161	.87248	.82572
.75	1.04091	.96055	.88585	.81644	.75197
1.00	1.05423	.94832	.85215	.76489	.68580
1.25	1.06739	.93650	.82036	.71743	.62633
1.50	1.08041	.92508	.79035	.67367	.57282
1.75	1.09328	.91404	.76198	.63328	.52461
2.00	1.10601	.90335	.73515	.59595	.48111
2.50	1.13108	.88299	.68568	.52943	.40625
3.00	1.15564	.86385	.64117	.47222	.34483
3.50	1.17972	.84584	.60101	.42281	.29418
4.00	1.20335	.82884	.56464	.37996	.25218
4.50	1.22654	.81278	.53161	.34264	.21720
5.00	1.24933	.79758	.50153	.31002	.18792
5.50	1.27172	.78316	.47405	.28138	.16329
6.00	1.29375	.76946	.44889	.25616	.14248
6.50	1.31542	.75642	.42579	.23387	.12482
7.00	1.33675	.74401	.40453	.21409	.10977
7.50	1.35776	.73216	.38492	.19650	.096889
8.00	1.37846	.72085	.36680	.18080	.085822
8.50	1.39887	.71002	.35000	.16675	.076277
9.00	1.41899	.69966	.33442	.15413	.068012
9.50	1.43884	.68972	.31992	.14277	.060831
10.00	1.45842	.68019	.30641	.13252	.054570
20.00	1.80681	.54573	.15506	.040964	.0099007
30.00	2.09872	.46818	.096776	.018274	.0030910
40.00	2.35493	.41632	.067554	.0099066	.0012854
50.00	2.58596	.37854	.050535	.0060535	.00063644
60.00	2.79803	.34946	.039620	.0040101	.00035424
70.00	2.99515	.32619	.032134	.0028153	.00021439
80.00	3.18008	.30702	.026738	.0020649	.00013817
90.00	3.35485	.29087	.022699	.0015672	$.93511 \cdot 10^{-4}$
100.00	3.52097	.27702	.019583	.0012224	$.65811 \cdot 10^{-4}$

Table 9. (Cont.) $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 9$.

p	${}_1F_1(\frac{9}{2};9;-p)$	${}_1F_1(\frac{11}{2};9;-p)$	${}_1F_1(\frac{13}{2};9;-p)$	${}_1F_1(\frac{15}{2};9;-p)$
0	1.0	1.0	1.0	1.0
.25	.88319	.85896	.83533	.81229
.50	.78124	.73892	.69867	.66041
.75	.69214	.63662	.58514	.53742
1.00	.61415	.54932	.49070	.43775
1.25	.54580	.47472	.41206	.35692
1.50	.48581	.41088	.34650	.29130
1.75	.43308	.35619	.29178	.23800
2.00	.38666	.30926	.24606	.19466
2.50	.30963	.23424	.17574	.13065
3.00	.24944	.17854	.12627	.088096
3.50	.20214	.13695	.091286	.059696
4.00	.16477	.10572	.066408	.040662
4.50	.13507	.082131	.048620	.027849
5.00	.11134	.064207	.035829	.019183
5.50	.092273	.050508	.026578	.013293
6.00	.076875	.039977	.019847	.0092689
6.50	.064374	.031833	.014920	.0065052
7.00	.054172	.025500	.011292	.0045963
7.50	.045806	.020546	.0086029	.0032700
8.00	.038910	.016649	.0065982	.0023431
8.50	.033200	.013566	.0050942	.0016911
9.00	.028449	.011114	.0039587	.0012295
9.50	.024479	.0091531	.0030961	.00090063
10.00	.021147	.0075765	.0024368	.00066468
20.00	.0021382	.00039756	$.59584 \cdot 10^{-4}$	$.6238 \cdot 10^{-5}$
30.00	.00045528	$.55870 \cdot 10^{-4}$	$.52901 \cdot 10^{-5}$	$.32827 \cdot 10^{-6}$
40.00	.00014314	$.13053 \cdot 10^{-4}$	$.89934 \cdot 10^{-6}$	$.39540 \cdot 10^{-7}$
50.00	$.56909 \cdot 10^{-4}$	$.41240 \cdot 10^{-5}$	$.22306 \cdot 10^{-6}$	$.75889 \cdot 10^{-8}$
60.00	$.26449 \cdot 10^{-4}$	$.15894 \cdot 10^{-5}$	$.70726 \cdot 10^{-7}$	$.19620 \cdot 10^{-8}$
70.00	$.13737 \cdot 10^{-4}$	$.70490 \cdot 10^{-6}$	$.26639 \cdot 10^{-7}$	$.62377 \cdot 10^{-9}$
80.00	$.77526 \cdot 10^{-5}$	$.34707 \cdot 10^{-6}$	$.11397 \cdot 10^{-7}$	$.23086 \cdot 10^{-9}$
90.00	$.46663 \cdot 10^{-5}$	$.18526 \cdot 10^{-6}$	$.53781 \cdot 10^{-8}$	$.95985 \cdot 10^{-10}$
100.00	$.29568 \cdot 10^{-5}$	$.10544 \cdot 10^{-6}$	$.27430 \cdot 10^{-8}$	$.43752 \cdot 10^{-10}$

Table 9. (Cont.) $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 9$.

p	${}_1F_1(\frac{17}{2};9;-p)$	${}_1F_1(\frac{19}{2};9;-p)$	${}_1F_1(\frac{21}{2};9;-p)$	${}_1F_1(\frac{23}{2};9;-p)$
0	1.0	1.0	1.0	1.0
.25	.78982	.76792	.74655	.72573
.50	.62404	.58947	.55662	.52542
.75	.49322	.45230	.41444	.37943
1.00	.38998	.34691	.30812	.27324
1.25	.30846	.26595	.22872	.19617
1.50	.24408	.20379	.16949	.14038
1.75	.19322	.15608	.12537	.10009
2.00	.15303	.11947	.092552	.071078
2.50	.096117	.069879	.050108	.035354
3.00	.060493	.040766	.026854	.017194
3.50	.038156	.023712	.014216	.0081205
4.00	.024125	.013745	.0074112	.0036831
4.50	.015294	.0079356	.0037894	.0015730
5.00	.0097236	.0045602	.0018884	.00060744
5.50	.0062018	.0026060	.00090800	.00018981
6.00	.0039694	.0014792	.00041391	$+.25204 \cdot 10^{-4}$
6.50	.0025503	.00083271	.00017256	$-.28569 \cdot 10^{-4}$
7.00	.0016455	.00046395	$.59908 \cdot 10^{-4}$	$-.37751 \cdot 10^{-4}$
7.50	.0010666	.00025510	$+.10995 \cdot 10^{-4}$	$-.31730 \cdot 10^{-4}$
8.00	.00069482	.00013783	$-.75533 \cdot 10^{-5}$	$-.22567 \cdot 10^{-4}$
8.50	.00045514	$.72702 \cdot 10^{-4}$	$-.12476 \cdot 10^{-4}$	$-.14544 \cdot 10^{-4}$
9.00	.00029993	$.37039 \cdot 10^{-4}$	$-.11887 \cdot 10^{-4}$	$-.86876 \cdot 10^{-5}$
9.50	.00019893	$.17872 \cdot 10^{-4}$	$-.95296 \cdot 10^{-5}$	$-.48221 \cdot 10^{-5}$
10.00	.00013287	$+.78350 \cdot 10^{-5}$	$-.69932 \cdot 10^{-5}$	$-.24513 \cdot 10^{-5}$
20.00	$.27 \cdot 10^{-6}$	$-.1 \cdot 10^{-7}$		
30.00	$.75370 \cdot 10^{-8}$	$-.197 \cdot 10^{-9}$	$+.19 \cdot 10^{-10}$	
40.00	$.62061 \cdot 10^{-9}$	$-.10529 \cdot 10^{-10}$	$.5869 \cdot 10^{-12}$	$-.610 \cdot 10^{-13}$
50.00	$.90575 \cdot 10^{-10}$	$-.11431 \cdot 10^{-11}$	$.46039 \cdot 10^{-13}$	$-.33126 \cdot 10^{-14}$
60.00	$.18896 \cdot 10^{-10}$	$-.19010 \cdot 10^{-12}$	$.60138 \cdot 10^{-14}$	$-.33369 \cdot 10^{-15}$
70.00	$.50362 \cdot 10^{-11}$	$-.42142 \cdot 10^{-13}$	$.10989 \cdot 10^{-14}$	$-.49733 \cdot 10^{-16}$
80.00	$.16045 \cdot 10^{-11}$	$-.11496 \cdot 10^{-13}$	$.25509 \cdot 10^{-15}$	$-.97567 \cdot 10^{-17}$
90.00	$.58565 \cdot 10^{-12}$	$-.36687 \cdot 10^{-14}$	$.70869 \cdot 10^{-16}$	$-.23482 \cdot 10^{-17}$
100.00	$.23790 \cdot 10^{-12}$	$-.13241 \cdot 10^{-14}$	$.22649 \cdot 10^{-16}$	$-.66211 \cdot 10^{-18}$

Table 10. $\alpha = -1/2 \longrightarrow 25/2$; $\beta = 10$.

p	${}_1F_1(-\frac{1}{2};10;-p)$	${}_1F_1(\frac{1}{2};10;-p)$	${}_1F_1(\frac{3}{2};10;-p)$	${}_1F_1(\frac{5}{2};10;-p)$
0	1.0	1.0	1.0	1.0
.25	1.01243	.98771	.96354	.93991
.50	1.02472	.97582	.92906	.88435
.75	1.03688	.96432	.89643	.83293
1.00	1.04891	.95319	.86552	.78528
1.25	1.06081	.94240	.83622	.74108
1.50	1.07259	.93194	.80841	.70004
1.75	1.08425	.92180	.78200	.66189
2.00	1.09580	.91197	.75690	.62639
2.50	1.11856	.89314	.71031	.56250
3.00	1.14089	.87537	.66804	.50686
3.50	1.16282	.85856	.62956	.45821
4.00	1.18436	.84263	.59446	.41552
4.50	1.20554	.82752	.56235	.37793
5.00	1.22637	.81315	.53289	.34472
5.50	1.24687	.79947	.50581	.31528
6.00	1.26705	.78644	.48085	.28910
6.50	1.28692	.77399	.45780	.26574
7.00	1.30651	.76210	.43647	.24485
7.50	1.32581	.75072	.41669	.22610
8.00	1.34485	.73982	.39831	.20924
8.50	1.36363	.72937	.38120	.19404
9.00	1.38216	.71933	.36524	.18029
9.50	1.40046	.70969	.35034	.16782
10.00	1.41852	.70041	.33640	.15650
20.00	1.74158	.56749	.17580	.051342
30.00	2.01401	.48916	.11142	.023550
40.00	2.25394	.43619	.078471	.012971
50.00	2.47077	.39734	.059041	.0080067
60.00	2.67010	.36728	.046476	.0053416
70.00	2.85557	.34315	.037807	.0037695
80.00	3.02972	.32322	.031531	.0027758
90.00	3.19441	.30640	.026817	.0021132
100.00	3.35102	.29195	.023170	.0016525

Table 10. (Cont.) $\alpha = -1/2 \longrightarrow 25/2$; $\beta = 10$.

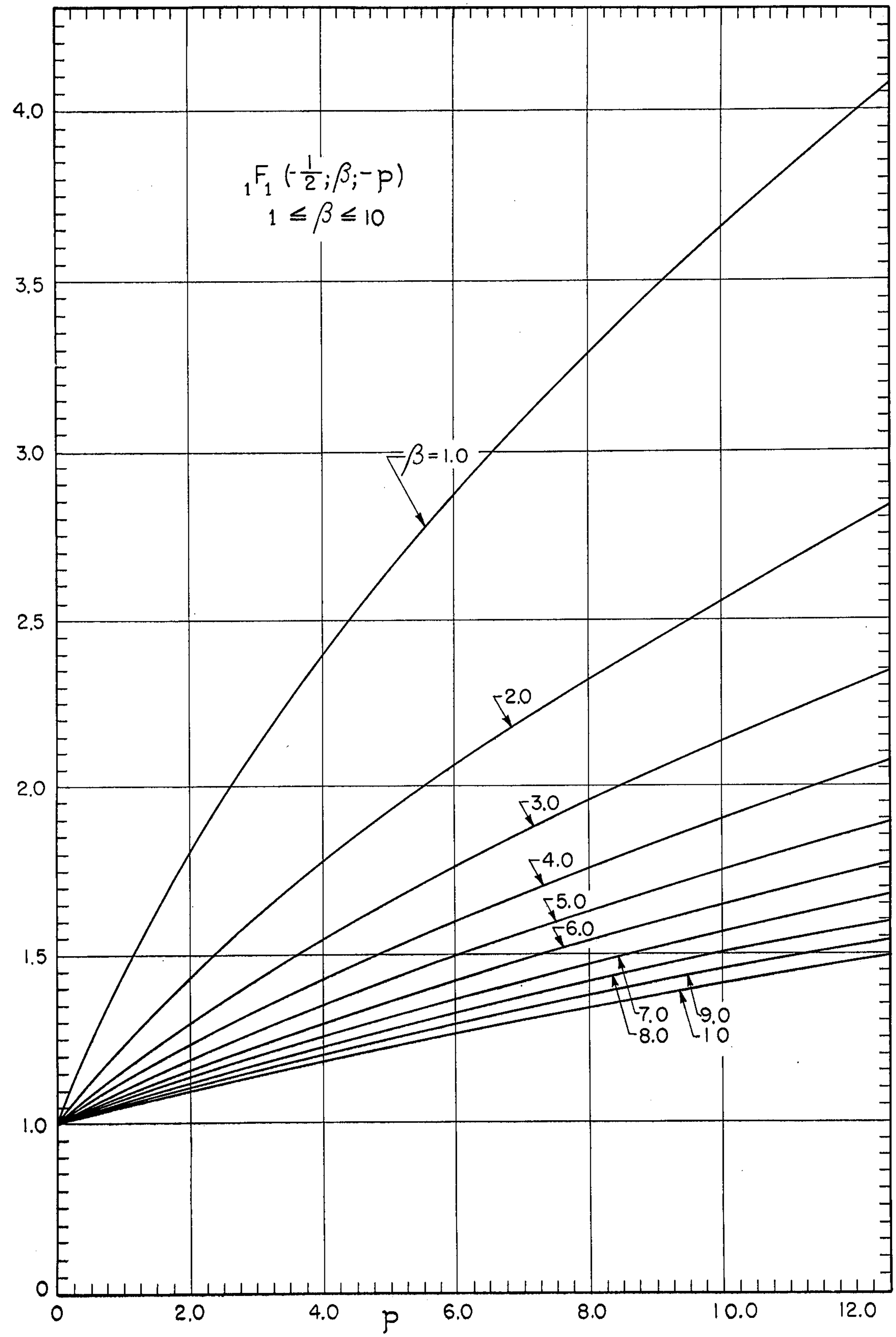
p	${}_1F_1(\frac{7}{2};10;-p)$	${}_1F_1(\frac{9}{2};10;-p)$	${}_1F_1(\frac{11}{2};10;-p)$	${}_1F_1(\frac{13}{2};10;-p)$
0	1.0	1.0	1.0	1.0
.25	.91681	.89422	.87215	.85057
.50	.84161	.80076	.76172	.72441
.75	.77356	.71806	.66621	.61779
1.00	.71189	.64480	.58351	.52756
1.25	.65594	.57980	.51180	.45112
1.50	.60511	.52207	.44954	.38628
1.75	.55889	.47073	.39542	.33122
2.00	.51681	.42501	.34831	.28440
2.50	.44342	.34784	.27142	.21057
3.00	.38216	.28618	.21271	.15680
3.50	.33078	.23665	.16763	.11743
4.00	.28750	.19668	.13285	.088456
4.50	.25089	.16426	.10587	.067022
5.00	.21978	.13784	.084835	.051080
5.50	.19325	.11620	.068343	.039158
6.00	.17052	.098403	.055348	.030195
6.50	.15099	.083692	.045056	.023419
7.00	.13413	.071480	.036864	.018268
7.50	.11954	.061300	.030311	.014332
8.00	.10685	.052776	.025043	.011307
8.50	.095791	.045611	.020788	.0089705
9.00	.086116	.039563	.017335	.0071555
9.50	.077626	.034439	.014519	.0057382
10.00	.070154	.030081	.012213	.0046257
20.00	.013978	.0034931	.00078329	.00015209
30.00	.0045549	.00079073	.00011982	$.15174 \cdot 10^{-4}$
40.00	.0019398	.00025700	$.29268 \cdot 10^{-4}$	$.27346 \cdot 10^{-5}$
50.00	.00097507	.00010432	$.95012 \cdot 10^{-5}$	$.70218 \cdot 10^{-6}$
60.00	.00054838	$.49169 \cdot 10^{-4}$	$.37290 \cdot 10^{-5}$	$.22779 \cdot 10^{-6}$
70.00	.00033441	$.25799 \cdot 10^{-4}$	$.16756 \cdot 10^{-5}$	$.87205 \cdot 10^{-7}$
80.00	.00021676	$.14672 \cdot 10^{-4}$	$.83312 \cdot 10^{-6}$	$.37763 \cdot 10^{-7}$
90.00	.00014736	$.88845 \cdot 10^{-5}$	$.44810 \cdot 10^{-6}$	$.17988 \cdot 10^{-7}$
100.00	.00010409	$.56569 \cdot 10^{-5}$	$.25662 \cdot 10^{-6}$	$.92431 \cdot 10^{-8}$

Table 10. (Cont.) $\alpha = -1/2 \longrightarrow 25/2$; $\beta = 10$.

p	${}_1F_1(\frac{15}{2};10;-p)$	${}_1F_1(\frac{17}{2};10;-p)$	${}_1F_1(\frac{19}{2};10;-p)$
0	1.0	1.0	1.0
.25	.82947	.80886	.78870
.50	.68878	.65474	.62223
.75	.57258	.53039	.49104
1.00	.47652	.43000	.38762
1.25	.39704	.34889	.30608
1.50	.33120	.28332	.24177
1.75	.27662	.23027	.19104
2.00	.23131	.18732	.15101
2.50	.16235	.12430	.094459
3.00	.11453	.082808	.059180
3.50	.081232	.055389	.037142
4.00	.057928	.037209	.023355
4.50	.041543	.025110	.014716
5.00	.029964	.017026	.0092940
5.50	.021739	.011603	.0058840
6.00	.015867	.0079493	.0037352
6.50	.0011651	.0054760	.0023782
7.00	.0086081	.0037939	.0015191
7.50	.0063994	.0026442	.00097376
8.00	.0047870	.0018543	.00062662
8.50	.0036033	.0013086	.00040494
9.00	.0027292	.00092959	.00026289
9.50	.0020800	.00066476	.00017153
10.00	.0015949	.00047862	.00011253
20.00	$.24005 \cdot 10^{-4}$	$.268 \cdot 10^{-5}$	$.13 \cdot 10^{-6}$
30.00	$.14885 \cdot 10^{-5}$	$.96221 \cdot 10^{-7}$	$.2320 \cdot 10^{-8}$
40.00	$.19346 \cdot 10^{-6}$	$.87568 \cdot 10^{-8}$	$.14200 \cdot 10^{-9}$
50.00	$.38785 \cdot 10^{-7}$	$.13497 \cdot 10^{-8}$	$.16509 \cdot 10^{-10}$
60.00	$.10314 \cdot 10^{-7}$	$.29146 \cdot 10^{-9}$	$.28630 \cdot 10^{-11}$
70.00	$.33449 \cdot 10^{-8}$	$.79552 \cdot 10^{-10}$	$.65293 \cdot 10^{-12}$
80.00	$.12562 \cdot 10^{-8}$	$.25791 \cdot 10^{-10}$	$.18180 \cdot 10^{-12}$
90.00	$.52821 \cdot 10^{-9}$	$.95399 \cdot 10^{-11}$	$.58932 \cdot 10^{-13}$
100.00	$.24293 \cdot 10^{-9}$	$.39163 \cdot 10^{-11}$	$.21531 \cdot 10^{-13}$

Table 10. (Cont.) $\alpha = -1/2 \longrightarrow 25/2$; $\beta = 10$.

p	${}_1F_1(\frac{21}{2};10;-p)$	${}_1F_1(\frac{23}{2};10;-p)$	${}_1F_1(\frac{25}{2};10;-p)$
0	1.0	1.0	1.0
.25	.76901	.74976	.73095
.50	.59119	.56156	.53328
.75	.45434	.42014	.38828
1.00	.34905	.31397	.28209
1.25	.26806	.23434	.20447
1.50	.20579	.17467	.14783
1.75	.15792	.13002	.10659
2.00	.12113	.096635	.076634
2.50	.071172	.053117	.039215
3.00	.041736	.028980	.019756
3.50	.024419	.015673	.0097624
4.00	.014251	.0083883	.0047060
4.50	.0082925	.0044327	.0021947
5.00	.0048094	.0023056	.00097662
5.50	.0027785	.0011752	.00040403
6.00	.0015979	.00058305	.00014648
6.50	.00091405	.00027849	$+.38183\cdot10^{-4}$
7.00	.00051949	.00012556	$-.2248\cdot10^{-5}$
7.50	.00029292	$.51270\cdot10^{-4}$	$-.13686\cdot10^{-4}$
8.00	.00016355	$.16891\cdot10^{-4}$	$-.13989\cdot10^{-4}$
8.50	$.90188\cdot10^{-4}$	$+.21905\cdot10^{-5}$	$-.10906\cdot10^{-4}$
9.00	$.48926\cdot10^{-4}$	$-.31995\cdot10^{-5}$	$-.74945\cdot10^{-5}$
9.50	$.25960\cdot10^{-4}$	$-.44597\cdot10^{-5}$	$-.47433\cdot10^{-5}$
10.00	$+.13345\cdot10^{-4}$	$-.40877\cdot10^{-5}$	$-.28070\cdot10^{-5}$
20.00	$-.9\cdot10^{-8}$		
30.00	$-.65\cdot10^{-10}$		$-.1\cdot10^{-11}$
40.00	$-.2501\cdot10^{-11}$	$+.1458\cdot10^{-12}$	$-.160\cdot10^{-13}$
50.00	$-.21405\cdot10^{-12}$	$.88832\cdot10^{-14}$	$-.66135\cdot10^{-15}$
60.00	$-.29418\cdot10^{-13}$	$.95212\cdot10^{-15}$	$-.54166\cdot10^{-16}$
70.00	$-.55596\cdot10^{-14}$	$.14768\cdot10^{-15}$	$-.68175\cdot10^{-17}$
80.00	$-.13220\cdot10^{-14}$	$.29795\cdot10^{-16}$	$-.11585\cdot10^{-17}$
90.00	$-.37396\cdot10^{-15}$	$.73217\cdot10^{-17}$	$-.24605\cdot10^{-18}$
100.00	$-.12121\cdot10^{-15}$	$.20980\cdot10^{-17}$	$-.62094\cdot10^{-19}$



$_1F_1(-\frac{1}{2};\beta;-p)$
$1 \leq \beta \leq 10$
$\beta = 1.0$
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10
0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
0
2.0
4.0
6.0
8.0
10.0
12.0
p

FIGURE 1

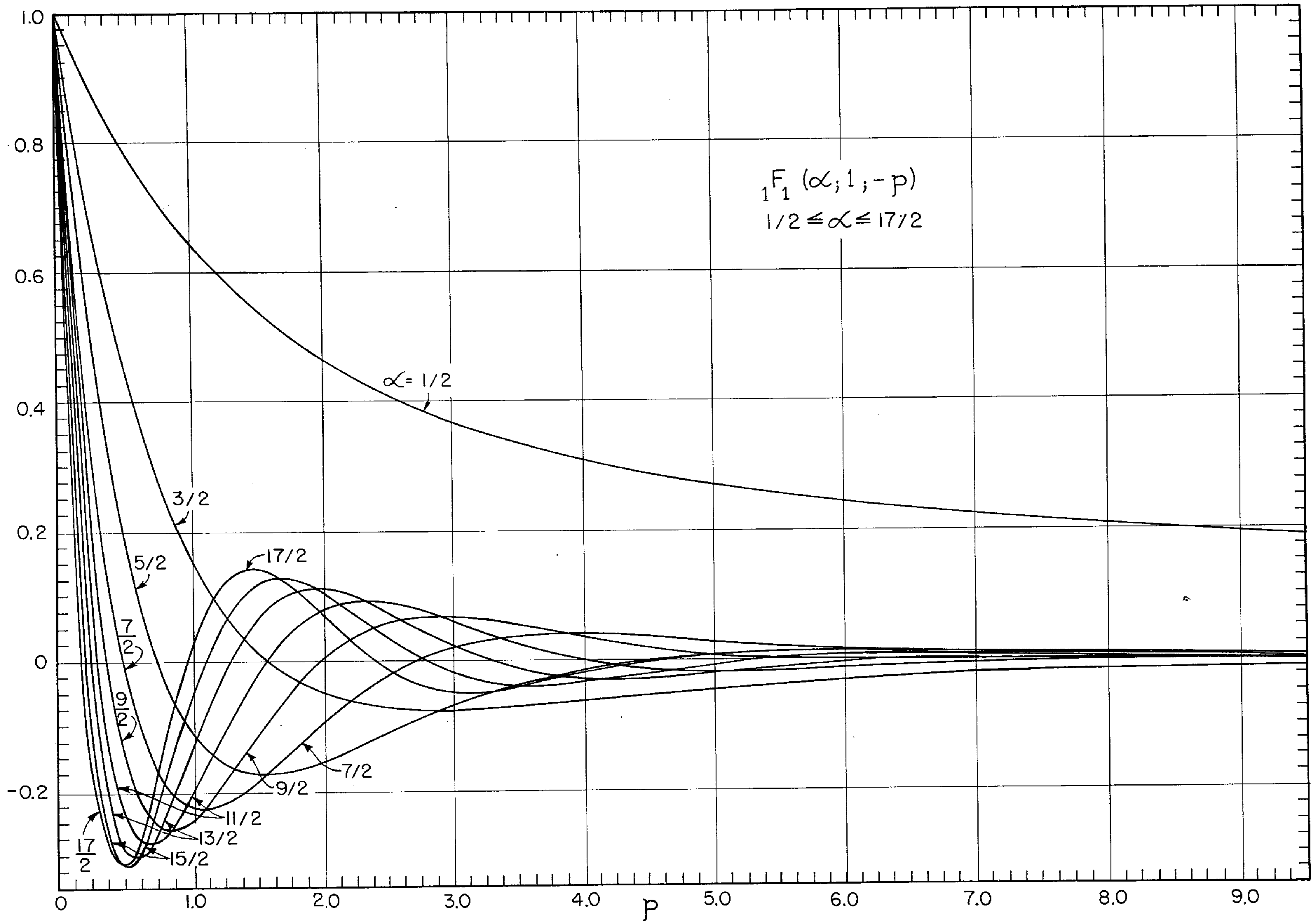
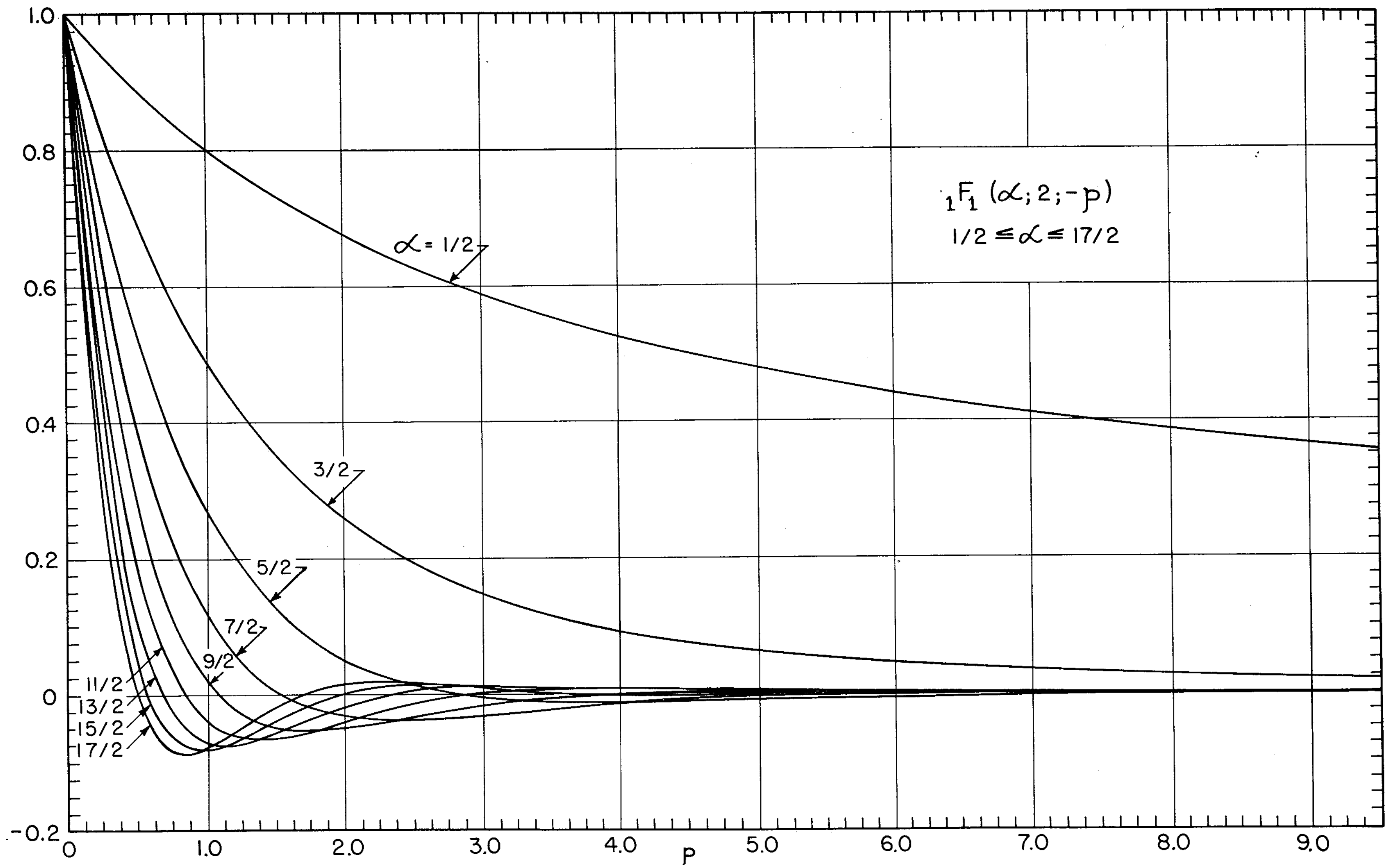
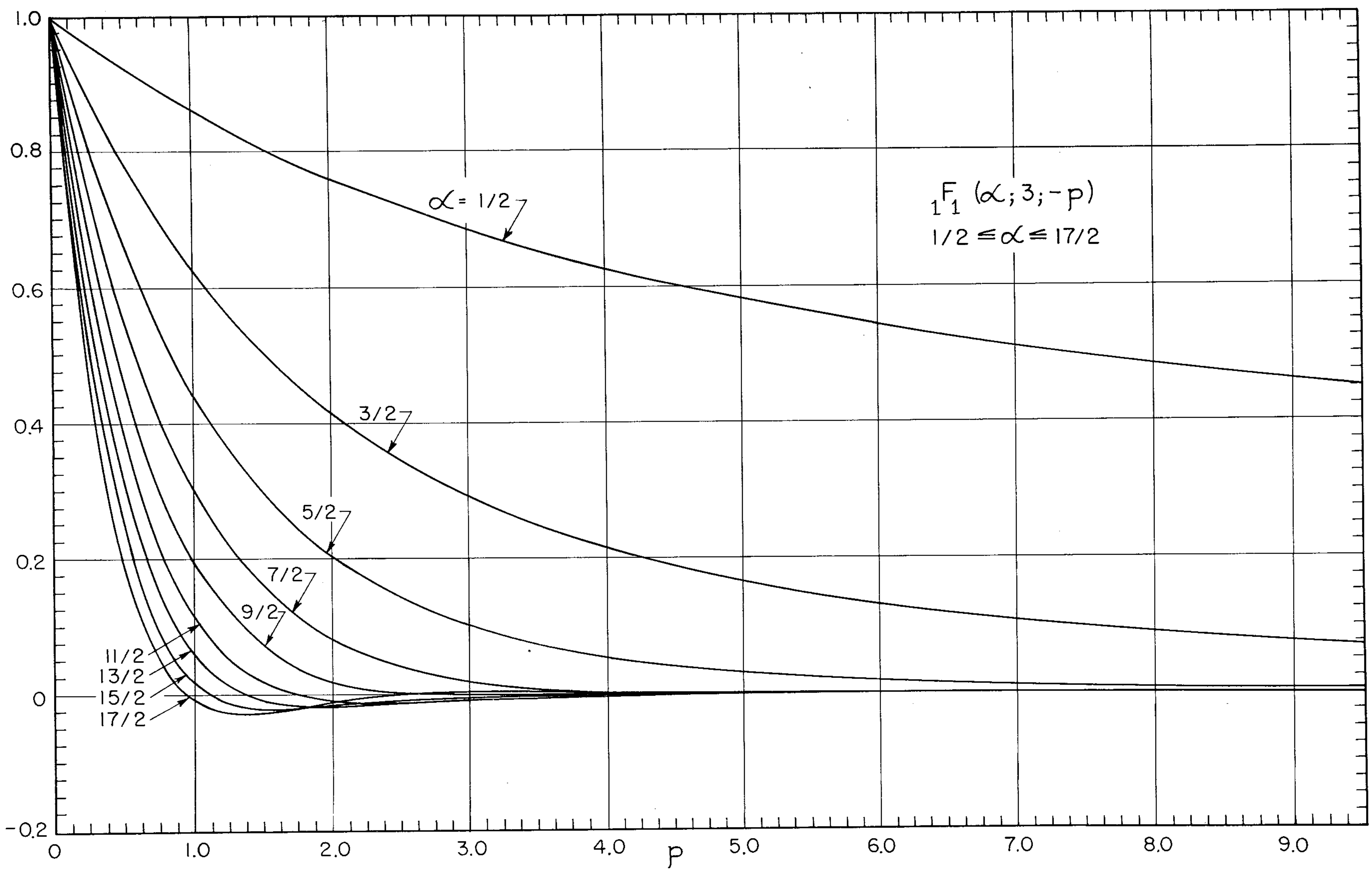

$_1F_1\ (\alpha; 1; -p)$
$1/2 \leq \alpha \leq 17/2$
$\alpha = 1/2$
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
p

FIGURE 2



$_1F_1\,(\alpha;2;-p)$
$1/2 \leq \alpha \leq 17/2$
$\alpha = 1/2$
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
P

FIGURE 3



${}_1F_1(\alpha;3;-p)$
$1/2 \leq \alpha \leq 17/2$
$\alpha = 1/2$
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
p

FIGURE 4

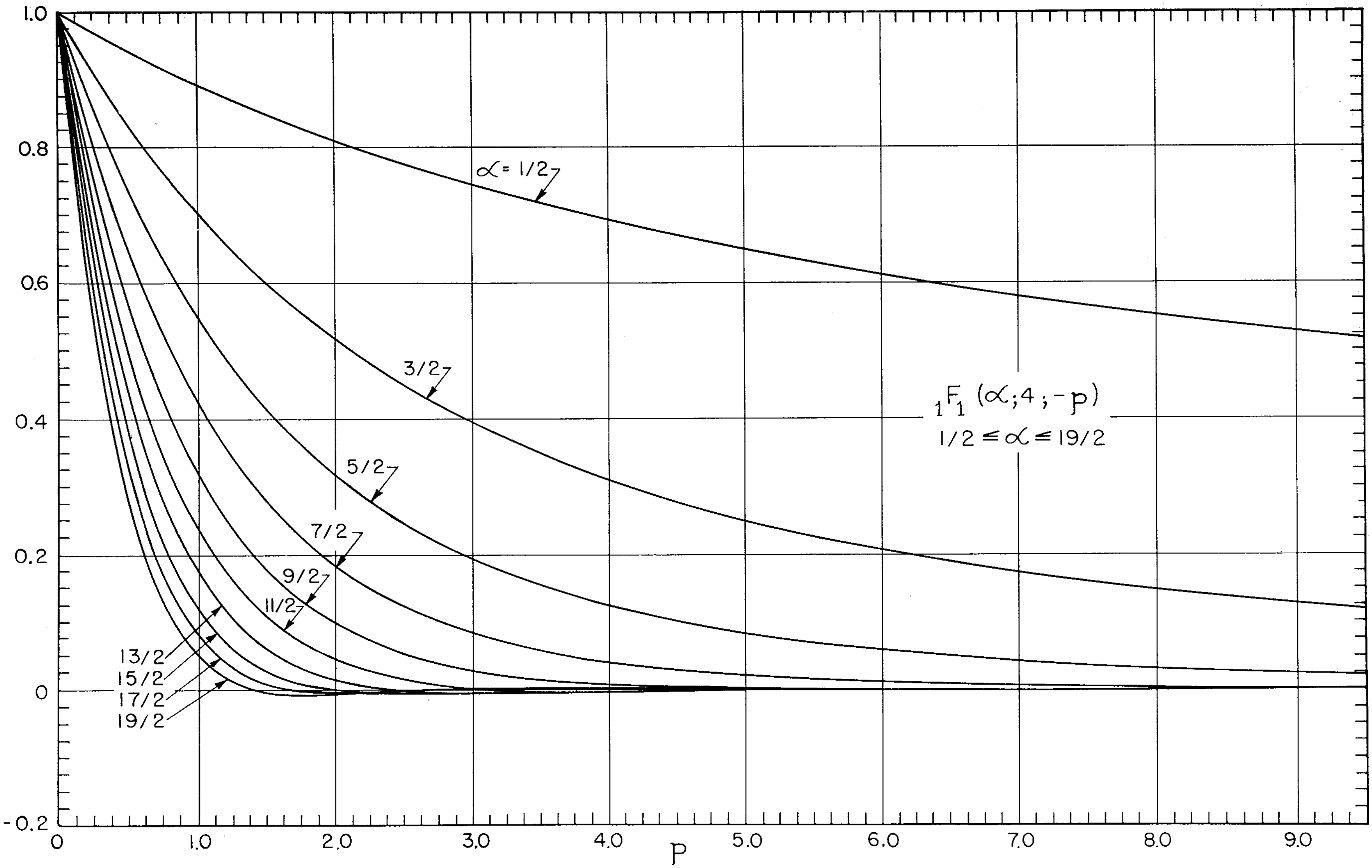
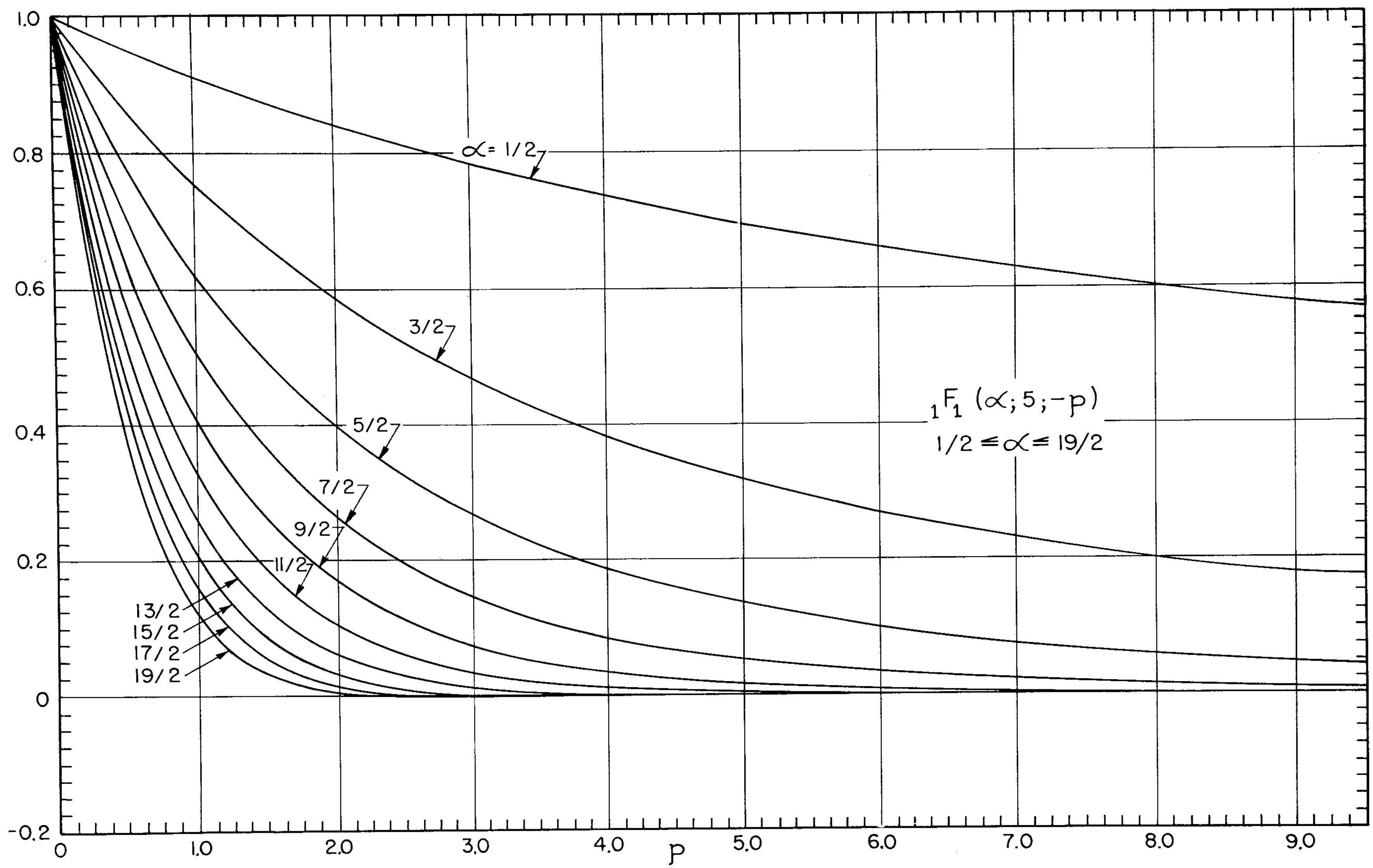
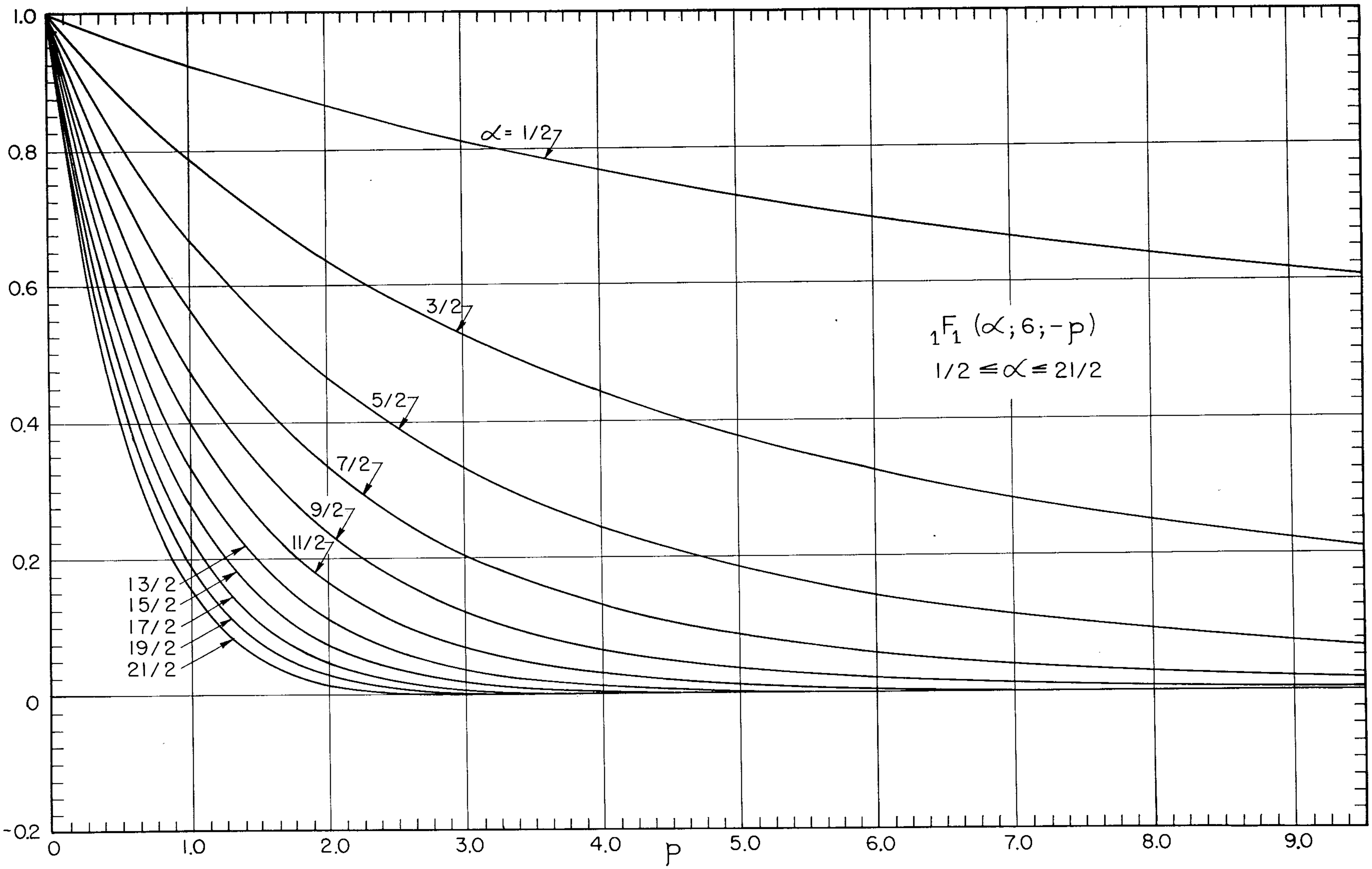

1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
P
α = 1/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
$_1F_1(\alpha; 4; -p)$
$1/2 \leq \alpha \leq 19/2$

FIGURE 5



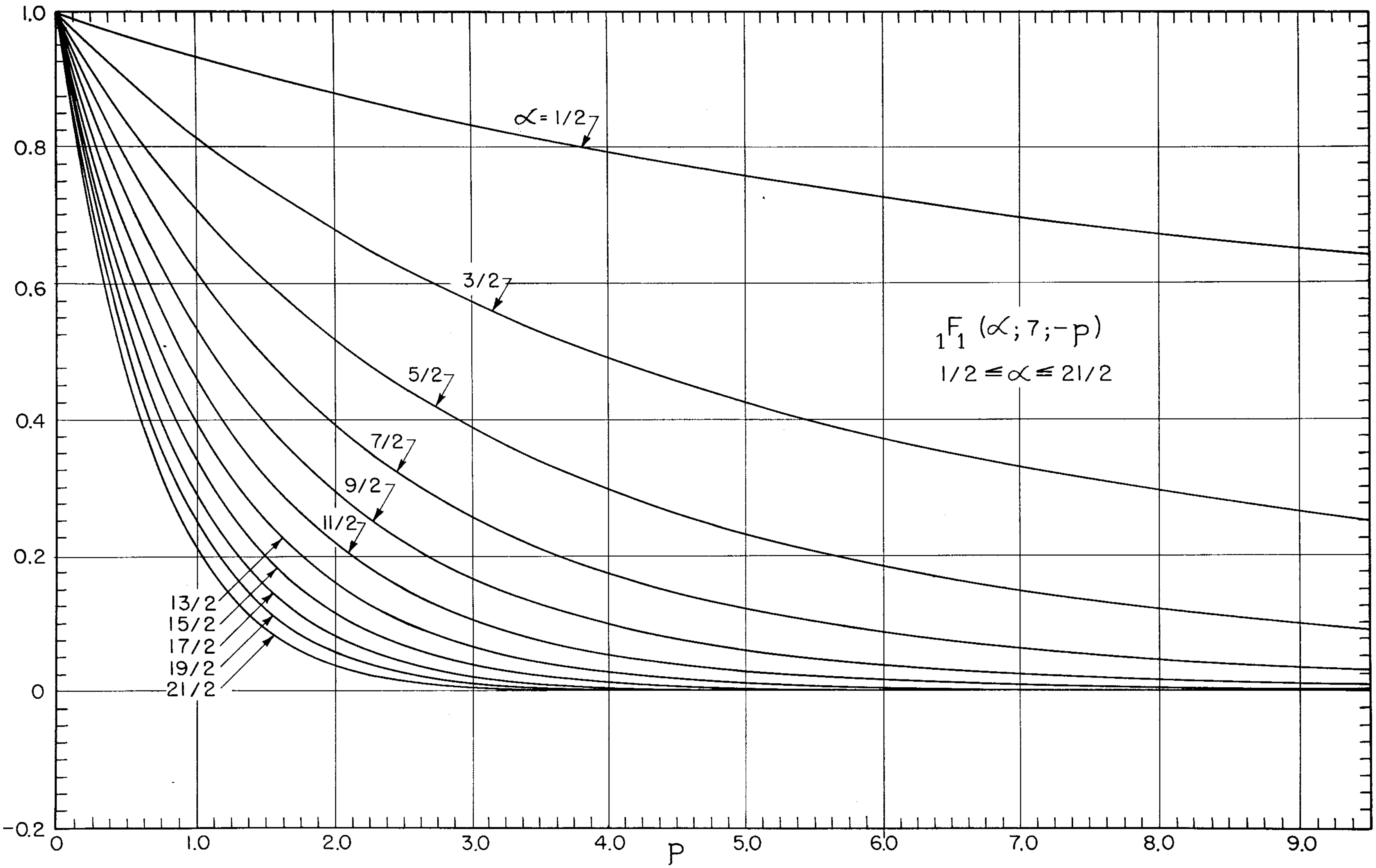
$_1F_1\ (\alpha; 5; -p)$
$1/2 \leq \alpha \leq 19/2$
$\alpha = 1/2$
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
p

FIGURE 6



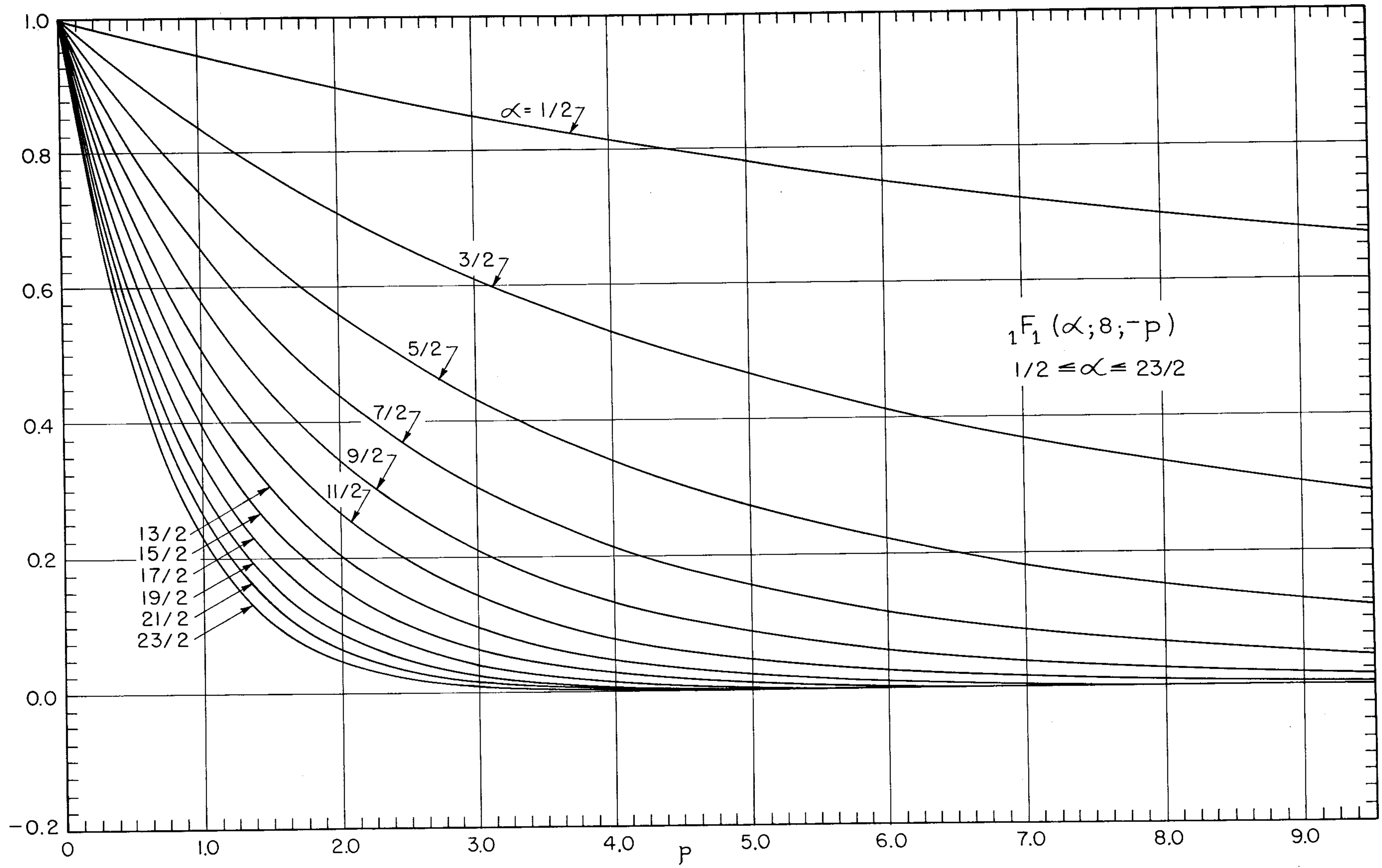
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
p
α = 1/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
21/2
$_1F_1$ (α; 6; -p)
1/2 ≤ α ≤ 21/2

FIGURE 7



1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
P
α = 1/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
21/2
$_1F_1\ (\alpha; 7; -p)$
$1/2 \leq \alpha \leq 21/2$

FIGURE 8



1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
P
α = 1/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
21/2
23/2
$_1F_1(\alpha; 8; -p)$
$1/2 \leq \alpha \leq 23/2$

FIGURE 9

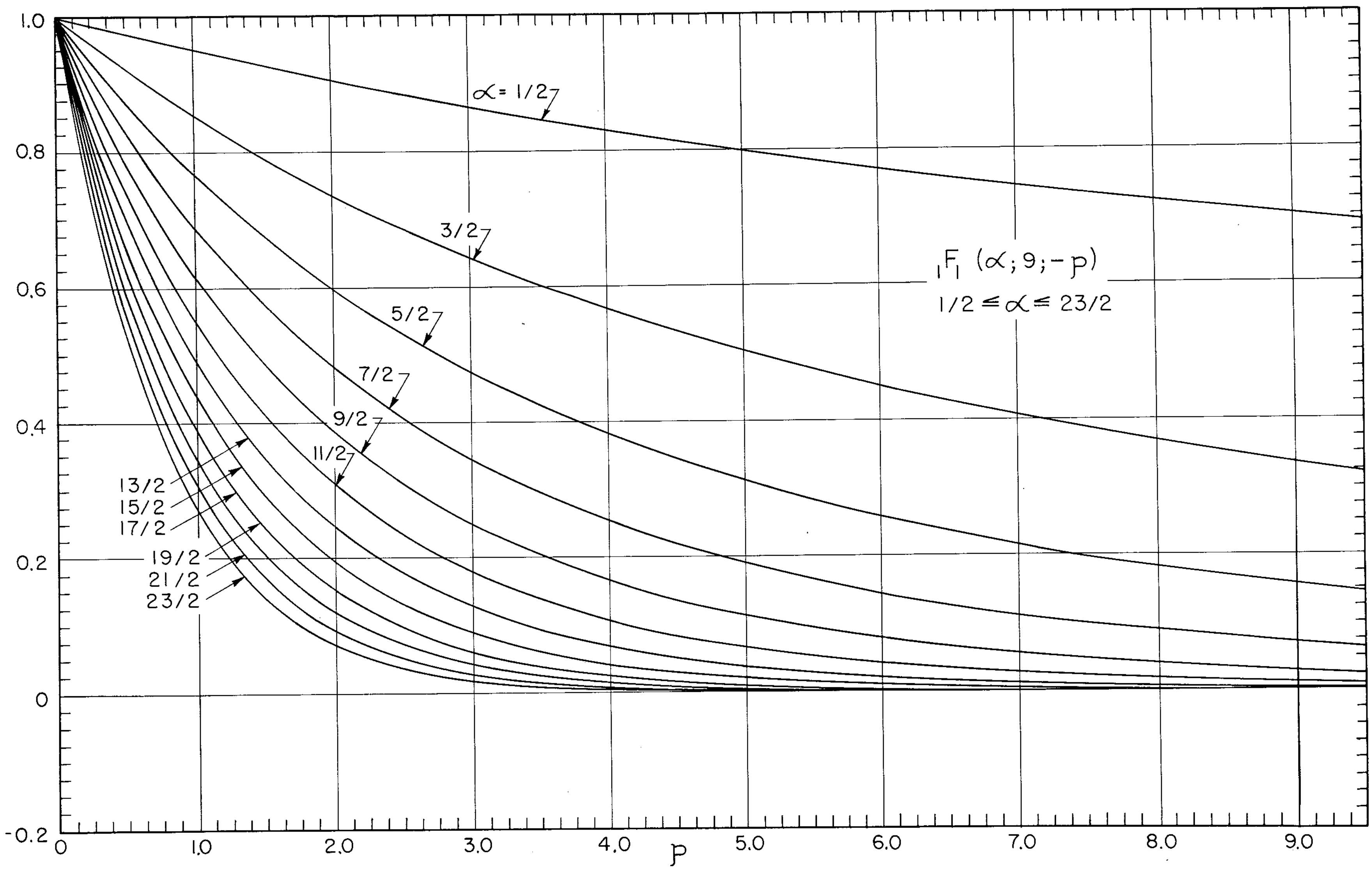
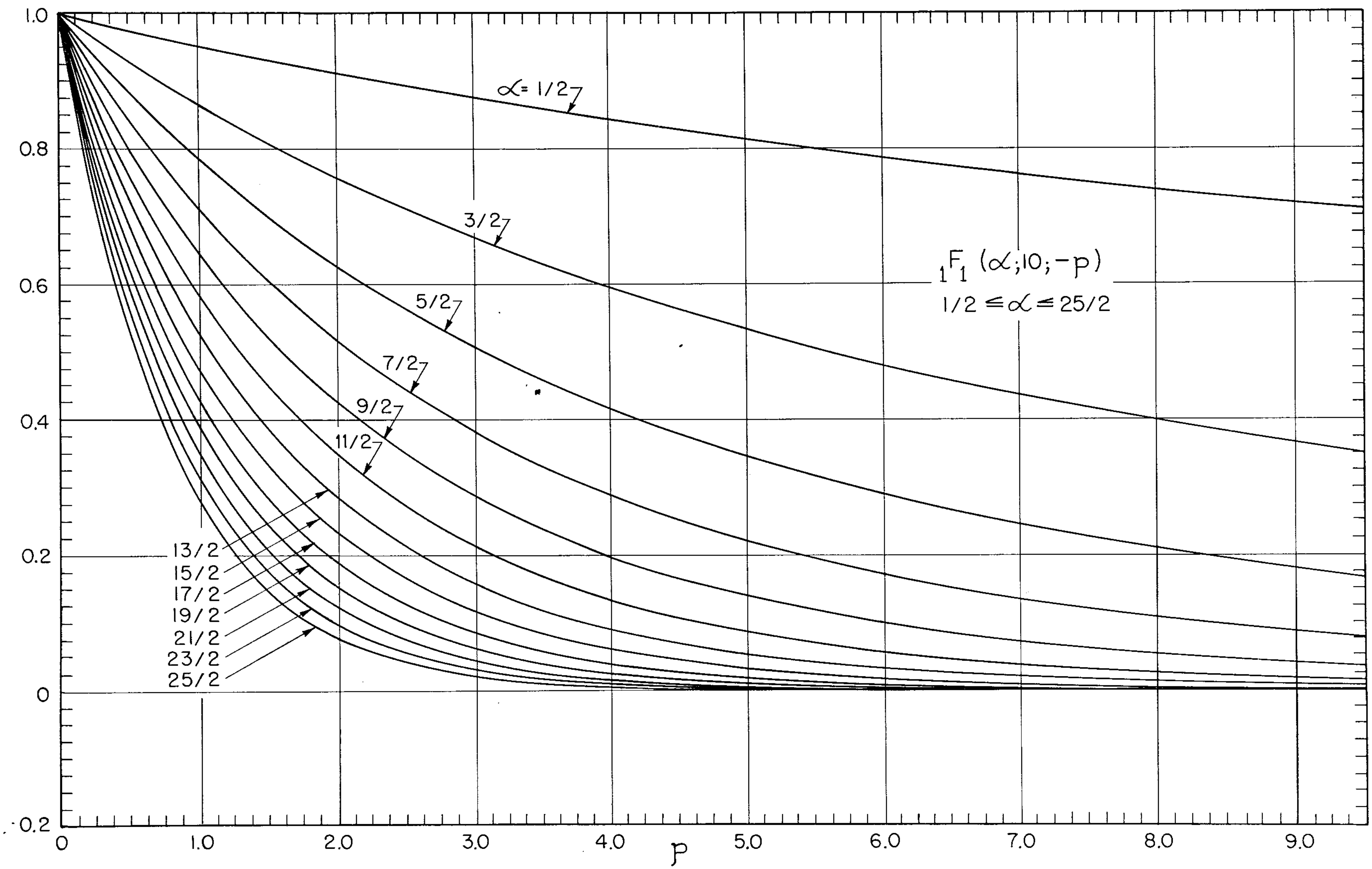

$_1F_1(\alpha; 9; -p)$
$1/2 \leq \alpha \leq 23/2$
$\alpha = 1/2$
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
21/2
23/2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
p

FIGURE 10



$_1F_1\,(\alpha;10;-p)$
$1/2 \leq \alpha \leq 25/2$
α = 1/2
3/2
5/2
7/2
9/2
11/2
13/2
15/2
17/2
19/2
21/2
23/2
25/2
1.0
0.8
0.6
0.4
0.2
0
-0.2
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
p

FIGURE 11

APPENDIX

Formulae and Calculations

Table 1. ${}_1F_1(\alpha;1;-p)$; $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 1$:

Here the first six functions are determined from tables of the modified Bessel functions I_o and I_1 according to

$$ {}_1F_1(-1/2;1;-p) = e^{-p/2}[(1+p)I_o(p/2)+pI_1(p/2)], \tag{A.1}$$

$$ {}_1F_1(1/2;1;-p) = e^{-p/2}\, I_o(p/2), \tag{A.2}$$

$$ {}_1F_1(3/2;1;-p) = e^{-p/2}[(1-p)I_o(p/2)+pI_1(p/2)], \tag{A.3}$$

$$ {}_1F_1(5/2;1;-p) = \frac{2e^{-p/2}}{3}[(p^2-3p+\tfrac{3}{2})I_o(p/2)+(2p-p^2)I_1(p/2)], \tag{A.4}$$

$$ {}_1F_1(7/2;1;-p) = \frac{2}{5}\, e^{-p/2}\left[\left\{\frac{-2p^3}{3}+\frac{14}{3}\,p^2-\frac{15}{2}\,p+\frac{5}{2}\right\} I_o(p/2) + \left\{\frac{2p^3}{3}-4p^2+\frac{23}{6}\,p\right\} I_1(p/2)\right], \tag{A.5}$$

$$ {}_1F_1(9/2;1;-p) = \frac{8}{105}\, e^{-p/2}\left[\left\{p^4-13p^3+47p^2-52.5p+13.125\right\} I_o(p/2) + \left\{-p^4+12p^3-35.5p^2+22p\right\} I_1(p/2)\right]. \tag{A.6}$$

For $\alpha = 11/2 \longrightarrow 17/2$, inclusive, the recurrence formula (1.8) is employed. The range of p for the above is $0 \leq p \leq 10.0$; for larger values of p, one uses the asymptotic expression (1.10). The mesh is as indicated in Tables 1-10.

Table 2. ${}_1F_1(\alpha;2;-p)$; $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 2$:

As before, the representation of ${}_1F_1$ in terms of the modified Bessel functions I_o and I_1 is used; the first five orders of ${}_1F_1$ (for $\beta = 2$) are

$${}_1F_1(-1/2;2;-p) = \frac{1}{3}\, e^{-p/2}[(3+2p)I_o(p/2)+(1+2p)I_1(p/2)], \qquad (A.7)$$

$${}_1F_1(1/2;2;-p) = e^{-p/2}[I_o(p/2)+I_1(p/2)], \qquad (A.8)$$

$${}_1F_1(3/2;2;-p) = e^{-p/2}[I_o(p/2)-I_1(p/2)], \qquad (A.9)$$

$${}_1F_1(5/2;2;-p) = e^{-p/2}[(1-\frac{2p}{3})I_o(p/2)-(\frac{1}{3}-\frac{2p}{3})I_1(p/2)], \qquad (A.10)$$

$${}_1F_1(7/2;2;-p) = \frac{2e^{-p/2}}{5}[(\frac{2p^2}{3}-3p+\frac{5}{2})I_o(\frac{p}{2})-(\frac{2p^2}{3}-\frac{7p}{3}+\frac{1}{2})I_1(\frac{p}{2})], \qquad (A.11)$$

for $0 \leq p \leq 10.0$. The higher orders, $\alpha = 9/2 \longrightarrow 17/2$, are computed with the help of the recurrence formula (1.8) with ${}_1F_1$ for $p \geq 10.0$ calculated from the asymptotic development (1.10).

Table 3. ${}_1F_1(\alpha;3;-p)$; $\alpha = -1/2 \longrightarrow 17/2$; $\beta = 3$:

The representation of ${}_1F_1$ for $\alpha = -1/2 \longrightarrow 9/2$ in terms of the modified Bessel functions becomes

$${}_1F_1(-1/2;3;-p) = \frac{4}{15p}\, e^{-p/2}[(2p^2+4p)I_o(p/2)+(2p^2+2p-1)I_1(p/2)], \qquad (A.12)$$

$${}_1F_1(1/2;3;-p) = \frac{4}{3}\, e^{-p/2}[I_o(p/2)-(\frac{1-p}{p})I_1(p/2)], \qquad (A.13)$$

$${}_1F_1(3/2;3;-p) = \frac{4}{p}\, e^{-p/2}\, I_1(p/2), \qquad (A.14)$$

$${}_1F_1(5/2;3;-p) = \frac{4}{3p}\, e^{-p/2}[p\, I_o(p/2)-(p+1)I_1(p/2)], \qquad (A.15)$$

$${}_1F_1(7/2;3;-p) = \frac{4}{15p}\, e^{-p/2}[(4p-2p^2)I_o(p/2)+(2p^2-2p-1)I_1(p/2)], \qquad (A.16)$$

$${}_1F_1(9/2;3;-p) = \frac{4}{105p}\, e^{-p/2}[(27p-24p^2+4p^3)I_o(p/2) -(3+9p-20p^2+4p^3)I_1(p/2)], \qquad (A.17)$$

for the indicated values of p for which $0 \leq p \leq 10.0$. When $\alpha = 11/2 \longrightarrow 17/2$, the recurrence formula (1.8) is used. As before, for larger values of p, ${}_1F_1$ is calculated with the help of (1.10).

Table 4: ${}_1F_1(\alpha;4;-p)$; $\alpha = -1/2 \longrightarrow 19/2$; $\beta = 4$:

Six functions are explicitly

$$ {}_1F_1(+1/2;4;-p) = \frac{4e^{-p/2}}{5p}\left[(2p-1)I_o(p/2)+\left(\frac{2p^2-3p+4}{p}\right)I_1(p/2)\right], \quad (A.18)$$

$$ {}_1F_1(3/2;4;-p) = \frac{4e^{-p/2}}{p}\left[I_o(p/2)+\left(1-\frac{4}{p}\right)I_1(p/2)\right], \quad (A.19)$$

$$ {}_1F_1(5/2;4;-p) = \frac{4}{p}e^{-p/2}\left[-I_o(p/2)+\left(1+\frac{4}{p}\right)I_1(p/2)\right], \quad (A.20)$$

$$ {}_1F_1(7/2;4;-p) = \frac{4}{5p}e^{-p/2}\left[(1+2p)I_o(p/2)-\left(2p+\frac{4}{p}+3\right)I_1(p/2)\right], \quad (A.21)$$

$$ {}_1F_1(9/2;4;-p) = \frac{4}{35}e^{-p/2}\left[\left(10-4p+\frac{1}{p}\right)I_o\left(\frac{p}{2}\right)+\left(4p-\frac{5}{p}-\frac{4}{p^2}-6\right)I_1\left(\frac{p}{2}\right)\right], \quad (A.22)$$

$$ {}_1F_1(11/2;4;-p) = \frac{4}{315}e^{-p/2}\left[\left(8p^2-60p+\frac{3}{p}+84\right)I_o(p/2) - \left(8p^2-52p+\frac{21}{p}+\frac{12}{p^2}+36\right)I_1(p/2)\right]. \quad (A.23)$$

Here ${}_1F_1(-1/2;4;-p)$ was determined from (A.18) and (A.19) by the recurrence relation (1.8). For $0 \leq p \leq 1.0$, the direct series expansion (1.2) was used, for $\alpha = 1/2 \longrightarrow 11/2$. The above were used for $1.25 < p < 10.0$.

Table 5. ${}_1F_1(\alpha;5;-p)$; $\alpha = -1/2 \longrightarrow 19/2$; $\beta = 5$:

Six representations of ${}_1F_1$ for a number of values of α and $\beta = 5$ are

$${}_1F_1(1/2;5;-p) = \frac{32e^{-p/2}}{35p^2}[(2p^2-2p+3)I_o(p/2)+(2p^2-4p-\frac{12}{p}+8)I_1(p/2)], \tag{A.24}$$

$${}_1F_1(3/2;5;-p) = \frac{32e^{-p/2}}{5p^2}[(p-3)I_o(p/2)+(p+\frac{12}{p}-4)I_1(p/2)], \tag{A.25}$$

$${}_1F_1(5/2;5;-p) = \frac{32}{3p^2} e^{-p/2}[3I_o(p/2)-\frac{12}{p} I_1(p/2)], \tag{A.26}$$

$${}_1F_1(7/2;5;-p) = \frac{32}{5p^2} e^{-p/2}[-(p+3)I_o(p/2)+(p+\frac{12}{p}+4)I_1(p/2)], \tag{A.27}$$

$${}_1F_1(9/2;5;-p) = \frac{32}{35p^2} e^{-p/2}[(2p^2+2p+3)I_o(\tfrac{p}{2})-(2p^2+4p+\frac{12}{p}+8)I_1(\tfrac{p}{2})], \tag{A.28}$$

$${}_1F_1(11/2;5;-p)= \frac{32}{315p} e^{-p/2}[(-4p^2+12p+\frac{3}{p}+3)I_o(p/2)$$
$$+(4p^2-8p-\frac{12}{p^2}-\frac{12}{p}-9)I_1(p/2)]. \tag{A.29}$$

For $\alpha = 1/2 \longrightarrow 9/2$ and $0 \leq p \leq 1.00$ the series form (1.2) was used to calculate ${}_1F_1$ while for $\alpha = 1/2 \longrightarrow 11/2$ the above results were applied, when $p \geq 1.25$. Recurrence relations, cf. (1.8), were then used for the higher values of α. The function ${}_1F_1(-1/2;5;-p)$ was also determined from previous calculations with the help of (1.8).

<u>Table 6</u>. ${}_1F_1(\alpha;6;-p)$; $\alpha = -1/2 \longrightarrow 21/2$; $\beta = 6$:

Six functions expressed in terms of I_o and I_1 from which calculations were made are

$${}_1F_1(3/2;6;-p) = \frac{32}{7p^2} e^{-p/2}[(2p+\frac{24}{p}-9)I_o(\tfrac{p}{2})+(2p+\frac{36}{p}-\frac{96}{p^2}-11)I_1(\tfrac{p}{2})], \tag{A.30}$$

$${}_1F_1(5/2;6;-p) = \frac{32e^{-p/2}}{p^2}[(1-\frac{8}{p})I_o(p/2)+(\frac{32}{p^2}-\frac{4}{p}+1)I_1(p/2)], \tag{A.31}$$

$$ {}_1F_1(7/2;6;-p) = \frac{32e^{-p/2}}{p^2}[(1+\frac{8}{p})I_o(p/2)-(\frac{32}{p^2}+\frac{4}{p}+1)I_1(p/2)], \tag{A.32}$$

$$ {}_1F_1(9/2;6;-p) = \frac{32}{7p^2}e^{-p/2}[-(2p+\frac{24}{p}+9)I_o(\frac{p}{2})+(\frac{96}{p^2}+\frac{36}{p}+2p+11)I_1(\frac{p}{2})], \tag{A.33}$$

$$ {}_1F_1(11/2;6;-p)= \frac{32}{63p^2}e^{-p/2}[(4p^2+6p+\frac{24}{p}+15)I_o(\frac{p}{2}) -(4p^2+10p+\frac{96}{p^2}+\frac{60}{p}+27)I_1(\frac{p}{2})], \tag{A.34}$$

$$ {}_1F_1(13/2;6;-p)= \frac{640}{3465p}e^{-p/2}[(-2p^2+\frac{5.25}{p}+\frac{6}{p^2}+7p+3)I_o(\frac{p}{2}) +(2p^2-5p-\frac{12.75}{p}-\frac{21}{p^2}-\frac{24}{p^3}-7)I_1(\frac{p}{2})], \tag{A.35}$$

and as before these are used for $1.25 \leq p \leq 10.0$. When $\alpha = 3/2 \longrightarrow 11/2$ the series form was used in the range $0 \leq p \leq 1.0$, and for all $0 \leq p \leq 10.0$, and $\alpha = -1/2, 1/2; 15/2 \longrightarrow 21/2$, one uses the recurrence relation (1.8).

<u>Table 7</u>. ${}_1F_1(\alpha;7;-p)$; $\alpha = -1/2 \longrightarrow 21/2$; $\beta = 7$.

The six functions for which calculations were made with the aid of tables of I_o and I_1 are here

$$ {}_1F_1(3/2;7;-p) = \frac{256}{21p^3}e^{-p/2}[(p^2-6p-\frac{60}{p}+24)I_o(\frac{p}{2}) +(p^2-7p-\frac{96}{p}+\frac{240}{p^2}+\frac{63}{2})I_1(\frac{p}{2})], \tag{A.36}$$

$$ {}_1F_1(5/2;7;-p) = \frac{384}{7p^3}e^{-p/2}[(\frac{40}{p}+p-8)I_o(\frac{p}{2})-(\frac{160}{p^2}-\frac{32}{p}-p+9)I_1(\frac{p}{2})], \tag{A.37}$$

$$ {}_1F_1(7/2;7;-p) = \frac{384}{p^3}e^{-p/2}[-\frac{8}{p}I_o(p/2)+(\frac{32}{p^2}+1)I_1(p/2)], \tag{A.38}$$

$$ {}_1F_1(9/2;7;-p) = \frac{384}{7p^3} e^{-p/2}[(p+\frac{40}{p}+8)I_o(\frac{p}{2})-(\frac{160}{p^2}+\frac{32}{p}+p+9)I_1(\frac{p}{2})], \quad (A.39) $$

$$ {}_1F_1(11/2;7;-p)= \frac{256}{21p^3} e^{-p/2}[-(p^2+6p+\frac{60}{p}+24)I_o(\frac{p}{2}) $$

$$ +(p^2+7p+\frac{96}{p}+\frac{240}{p^2}+\frac{63}{2})I_1(\frac{p}{2})], \quad (A.40) $$

$$ {}_1F_1(13/2;7;-p)= \frac{128}{231p^3} e^{-p/2}[(4p^3+8p^2+27p+\frac{120}{p}+72)I_o(\frac{p}{2}) $$

$$ -(4p^3+12p^2+41p+\frac{288}{p}+\frac{480}{p^2}+123)I_1(\frac{p}{2})]. \quad (A.41) $$

Here for $\alpha = 3/2 \longrightarrow 11/2$, the range of p using (A.36) - (A.41) above is $1.25 \leq p \leq 10.0$, and for $\alpha = 13/2$, $0 \leq p \leq 10.0$. For $\alpha = 3/2 \longrightarrow 11/2$, and $0 \leq p \leq 1.0$, the series expansion of ${}_1F_1$ was used. The recurrence relation (1.8) is then employed for $\alpha =$ $-1/2$, $1/2$; $15/2 \longrightarrow 21/2$, for all $0 \leq p \leq 10.0$.

<u>Table 8</u>. ${}_1F_1(\alpha;8;-p)$; $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 8$:

Here five functions for ${}_1F_1$ in terms of I_o and I_1 are given:

$$ {}_1F_1(5/2;8;-p) = \frac{256}{3p^3} e^{-p/2}[(p+\frac{60}{p}-\frac{240}{p^2}-\frac{21}{2})I_o(\frac{p}{2}) $$

$$ +(p+\frac{72}{p}-\frac{240}{p^2}+\frac{960}{p^3}-\frac{23}{2})I_1(\frac{p}{2})], \quad (A.42) $$

$$ {}_1F_1(7/2;8;-p) = \frac{384}{p^3} e^{-p/2}[(\frac{96}{p^2}-\frac{8}{p}+1)I_o(\frac{p}{2})-(\frac{16}{p}-\frac{32}{p^2}+\frac{384}{p^3}-1)I_1(\frac{p}{2})], \quad (A.43) $$

$$ {}_1F_1(9/2;8;-p) = \frac{384}{p^3} e^{-p/2}[-(\frac{8}{p}+\frac{96}{p^2}+1)I_o(\frac{p}{2})+(\frac{16}{p}+\frac{32}{p^2}+\frac{384}{p^3}+1)I_1(\frac{p}{2})], \quad (A.44) $$

$$ {}_1F_1(11/2;8;-p)= \frac{256}{3p^3} e^{-p/2}[(p+\frac{60}{p}+\frac{240}{p^2}+\frac{21}{2})I_o(\frac{p}{2}) $$

$$ -(p+\frac{72}{p}+\frac{240}{p^2}+\frac{960}{p^3}+\frac{23}{2})I_1(\frac{p}{2})], \quad (A-45) $$

$$ {}_1F_1(13/2;8;-p) = \frac{2560}{11p^3}\, e^{-p/2}\Big[-\Big(\frac{p^2}{15}+\frac{p}{2}+\frac{10}{p}+\frac{24}{p^2}+2.65\Big)I_o\Big(\frac{p}{2}\Big) $$

$$ +\Big(\frac{p^2}{15}+\frac{1.7p}{3}+\frac{13.6}{p}+\frac{40}{p^2}+\frac{96}{p^3}+3.25\Big)I_1\Big(\frac{p}{2}\Big)\Big]. \quad (A.46) $$

As above, the range of p for which (A.42) - (A.46) were used is $1.25 \leq p \leq 10.0$, while ${}_1F_1$ for $\alpha = 5/2 \longrightarrow 13/2$ is calculated by the series (1.2) for $0 \leq p \leq 1.0$. The remaining functions, $\alpha = -1/2$, $1/2$, $3/2$, and $15/2 \longrightarrow 23/2$ are determined from the recurrence relation (1.8).

Table 9. ${}_1F_1(\alpha;9;-p)$; $\alpha = -1/2 \longrightarrow 23/2$; $\beta = 9$:

The five functions ${}_1F_1$ expressed in terms of I_o and I_1 for $\beta = 9$ are here

$$ {}_1F_1(5/2;9;-p) = \frac{4096}{33p^4}\, e^{-p/2}\Big[\Big(p^2-13p-\frac{480}{p}+\frac{1680}{p^2}+97.5\Big)I_o\Big(\frac{p}{2}\Big) $$

$$ +\Big(p^2-14p-\frac{600}{p}+\frac{1920}{p^2}-\frac{6720}{p^3}+112\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.47) $$

$$ {}_1F_1(7/2;9;-p) = \frac{2048}{3p^4}\, e^{-p/2}\Big[\Big(p+\frac{96}{p}-\frac{672}{p^2}-15\Big)I_o\Big(\frac{p}{2}\Big) $$

$$ +\Big(p+\frac{144}{p}-\frac{384}{p^2}+\frac{2688}{p^3}-16\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.48) $$

$$ {}_1F_1(9/2;9;-p) = \frac{6144}{p^4}\, e^{-p/2}\Big[\Big(\frac{96}{p^2}+1\Big)I_o\Big(\frac{p}{2}\Big)-\Big(\frac{16}{p}+\frac{384}{p^3}\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.49) $$

$$ {}_1F_1(11/2;9;-p) = \frac{2048}{3p^4}\, e^{-p/2}\Big[-\Big(\frac{672}{p^2}+\frac{96}{p}+p+15\Big)I_o\Big(\frac{p}{2}\Big) $$

$$ +\Big(p+\frac{144}{p}+\frac{384}{p^2}+\frac{2688}{p^3}+16\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.50) $$

$$ {}_1F_1(13/2;9;-p) = \frac{4096}{33p^4}\, e^{-p/2}\Big[\Big(p^2+13p+\frac{480}{p}+\frac{1680}{p^2}+97.5\Big)I_o\Big(\frac{p}{2}\Big) $$

$$ -\Big(p^2+14p+\frac{600}{p}+\frac{1920}{p^2}+\frac{6720}{p^3}+112\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.51) $$

Here the range of p for which the above were used is $2.5 \leq p \leq 10.0$, while for $0 \leq p \leq 2.0$ the series form was employed. The cases of $\alpha = -1/2, 1/2, 3/2, 15/2 \longrightarrow 23/2$ were computed with the aid of recurrence formula (1.8) for p: $0 \leq p \leq 10.0$.

Table 10. ${}_1F_1(\alpha;10;-p)$; $\alpha = -1/2 \longrightarrow 25/2$; $\beta = 10$.

In this instance four functional expressions for ${}_1F_1$ were obtained:

$$ {}_1F_1(7/2;10;-p) = \frac{6144e^{-p/2}}{11p^4}\Big[\Big(2p + \frac{360}{p} - \frac{2016}{p^2} + \frac{10752}{p^3} - 37\Big)I_o\Big(\frac{p}{2}\Big) + \Big(2p + \frac{400}{p} - \frac{2784}{p^2} + \frac{8064}{p^3} - \frac{43008}{p^4} - 39\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.52) $$

$$ {}_1F_1(9/2;10;-p) = \frac{6144}{p^4}\, e^{-p/2}\Big[\Big(\frac{-24}{p} + \frac{96}{p^2} - \frac{1536}{p^3} + 1\Big)\, I_o\Big(\frac{p}{2}\Big) + \Big(\frac{-16}{p} + \frac{288}{p^2} - \frac{384}{p^3} + \frac{6144}{p^4} + 1\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.53) $$

$$ {}_1F_1(11/2;10;-p) = \frac{6144}{p^4}\, e^{-p/2}\Big[\Big(\frac{24}{p} + \frac{96}{p^2} + \frac{1536}{p^3} + 1\Big)I_o\Big(\frac{p}{2}\Big) - \Big(\frac{16}{p} + \frac{288}{p^2} + \frac{384}{p^3} + \frac{6144}{p^4} + 1\Big)I_1\Big(\frac{p}{2}\Big)\Big], \quad (A.54) $$

$$ {}_1F_1(13/2;10;-p) = \frac{6144}{11p^4}\, e^{-p/2}\Big[-\Big(2p + \frac{360}{p} + \frac{2016}{p^2} + \frac{10752}{p^3} + 37\Big)I_o\Big(\frac{p}{2}\Big) + \Big(2p + \frac{400}{p} + \frac{2784}{p^2} + \frac{8064}{p^3} + \frac{43008}{p^4} + 39\Big)I_1\Big(\frac{p}{2}\Big)\Big]. \quad (A.55) $$

The range of p here is $2.5 \leq p \leq 10.0$, while for $\alpha = 7/2 \longrightarrow 15/2$, the series development of ${}_1F_1$ was used for $0 \leq p \leq 2.0$. In all other cases, $\alpha = -1/2 \longrightarrow 5/2$; $\alpha = 15/2 \longrightarrow 25/2$, the recurrence relation (1.8) was employed for $0 \leq p \leq 10.0$. As usual, for $p \geq 20.0$, the asymptotic series (1.10) was used.

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